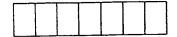
國立臺北科技大學

九十二學年度機電科技研究所博士班入學考試

控制系統(電機組)試題

填准考證號碼

第一頁 共一頁



注意事項:

- 1. 本試題共【五】題,配分共100分。
- 2. 請按順序標明題號作答,不必抄題。
- 3. 全部答案均須答在答案卷之答案欄內,否則不予計分。
- 1. Explain the following terms.
 - (a) Gain/Phase margin
- (5%)
- (b) Separation principle (5%)
- (c) Linear quadratic regulation (5%)
- (d) Equilibrium point
- (5%)

2. For the dynamic system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

where x(t) is the state vector and y(t) is the output. Prove that the state feedback law u(t) = -Kx(t) + r does not affect the controllability. (20%)

3. Show that the transformation $x = P\overline{x}$ converts the system $\dot{x} = Ax + Bu$ into

$$\dot{\overline{x}} = \overline{A}\overline{x} + \overline{B}u \text{ where } \overline{A} = P^{-1}AP = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_2 & -\alpha_1 \end{bmatrix},$$

$$\overline{B} = P^{-1}B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \text{ and } P = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} \begin{bmatrix} \alpha_{n-1} & \alpha_{n-2} & \cdots & \alpha_1 & 1 \\ \alpha_{n-2} & \alpha_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ \alpha_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}. (20\%)$$

4. For the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

- (a) Find the zero-input response of x(t) with the initial state $x_0 = x(0)$. (10%)
- (b) Design an observer that reconstructs the states and pick the observer roots to be at $s = -5 \pm j5$. (5%)
- (c) Design a state feedback law u(t) = -Kx(t) + r so that the closed-loop poles are at $s = -1 \pm j1$. (5%)
- 5. Consider the Van der Pol oscillator described by

$$\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2)$$

$$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2)$$

- (a) Find the equilibrium point. (10%)
- (b) Examine the stability using Lyapunov approach. (Hint: Consider the Lyapunov candidate $V(x) = x_1^2 + x_2^2$.) (10%)