國立臺北科技大學

九十二學年度機電整合研究所入學考試

工程數學試題

填准考證號碼　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共【10】題，配分共100分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Solve the initial value problem $xy' - y = \dfrac{y}{\ln y - \ln x}$; $y(2) = 2$. (10%)

2. Find the general solution of differential equation $y^{(4)} + 16y = t^2 + 1$ (10%)

3. Solve the initial value problem $y'' + 4y = f(t)$, $y(0) = 1$, $y'(0) = 0$ with

$$f(t) = \begin{cases} 0 & \text{for } 0 \le t < 4 \\ 3 & \text{for } t \ge 4 \end{cases}$$ (10%)

4. Find the first four nonzero terms of the power series solution of the initial value problem, about the point where the initial conditions are given

$$y'' + y' - xy = 0 \ ; \quad y(0) = -2 \ , \quad y'(0) = 0$$ (10%)

5. Show that the eigenvalues of $\begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix}$ are real. (α, β and γ are real numbers) (10%)

6. Find the general solution of system of linear differential equations

$$x_1' = -2x_1 + x_2$$
$$x_2' = -4x_1 + 3x_2 + 3$$ (10%)

7. Evaluate $\oint_c \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ with C the positively oriented square with vertices (1,1), (-1,1), (-1,-1) and (1,-1) by (10%)

(a) directly integrating along the curve. (hint: $\int \frac{1}{1+x^2}dx = \tan^{-1} x$)

(b) the use of Green's theorem.

8. Consider the level surface $\varphi(x, y, z) = z - \sqrt{x^2+y^2} = 2$ (10%)

(a) show the graph of this level surface in 3-D diagram.

(b) find the equation of tangent plane to this surface at point (3,4,7).

9. Periodic function $f(t)$ has period 2π and (10%)

$f(x) = x+\pi \qquad -\pi < x < \pi$

(a) find the Fourier series of $f(t)$.

(b) find the the value of this series at $x=\pi$

(c) use the result above to show that $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \cdots$

10. Wave equation be expressed as boundary value problem: (10%)

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 0 < x < \pi, \quad t > 0$$

$$u(0, t)=0, \quad u(\pi, t)=0 \qquad t > 0$$

$$u(x,0)= 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = x$$

Show that u(x,t) can be expressed as

$u(x, t)= \sum_{n=1}^{\infty}(A_n \cos nat + B_n \sin nat)\sin nx$ and find A_n and B_n.

(By assuming that $u(x, t)=$ X (x)T(t) and $\frac{X''}{X} = -\lambda^2$)