國立臺北科技大學

九十二學年度製造科技研究所入學考試

微分方程試題

填准考證號碼　　第一頁　共一頁

注意事項：

1. 本試題共三大題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Solve the following differential equations. In case the power series method is used, at least four (4) non-zero terms must be given. (@15% x 4 = 60%)

(a) $y'' - 8xy' + 16y = 3;\quad y(0) = y'(0) = 0$.

(b) Let $y(x) = x^2 - 1$ be a solution of the ODE

$(x^2+1)y'' - 2xy' + 2y = 0$.

Find its general solution.

(c) $y^{(4)} - y = -4\cos(x)$.

(d) Find $x(y)$ satisfies ODE $(xe^y - 1)y' + e^y = 0$.

2. Prove the Laplace transform of the Dirac delta function at a is exp($-as$), i.e., to prove

$$L[\delta(t-a)] = e^{-as}$$

where $L[.]$ denotes the Laplace transform from t to s. (20%)

3. Solve the initial value problem (20%)

$$\begin{cases} y_1'(t) = 3y_1 - y_2 + y_3 \\ y_2'(t) = y_1 + y_2 - y_3 \\ y_3'(t) = y_1 - y_2 + y_3 \end{cases} \text{ with the initial condition } \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix}(0) = \begin{Bmatrix} 1 \\ 5 \\ 1 \end{Bmatrix}.$$