國立臺北科技大學

九十二學年度工業工程與管理系碩士班入學考試

作業研究試題

填准考證號碼

注意事項：

1. 本試題共[redacted]題，配分共100分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. (30 points) Consider the following problem

$$\text{Maximize } Z = 2X_1 + 3X_2$$

$$\text{S.T.} \quad X_1 \le 4$$

$$X_2 \le 6$$

$$3X_1 + 2X_2 \le 18$$

$$X_1, X_2 \ge 0$$

Let S_1, S_2, and S_3, denote the slack variables for the respective constraints. After applying the simplex method, the final tableau is

B.V.	Eq.	Z	X_1	X_2	S_1	S_2	S_3	RHS
Z	(0)	1	0	0	0	5/3	2/3	22
S_1	(1)	0	0	0	1	2/3	-1/3	2
X_2	(2)	0	0	1	0	1	0	6
X_1	(3)	0	1	0	0	-2/3	1/3	2

Note: Do not use graphical method to solve the following problems.

(a) (10 points) What is the range of b_2 (resource of the second constraint, 6) so that the optimal solution remains the same?

(b) (10 points) What is the new optimal solution if an addition constraint, $2X_1 + X_2 \leq 8$ is added? (apply Gaussian elimination and dual simplex method to solve this question)

(c) (5 points) If the RHS changes from $[b_1,b_2,b_3]=[4,6,18]$ to $[b_1,b_2,b_3]=[2,9,16]$. Will the optimal solution remain the same?

(d) (5 points) Will the optimal solution remain the same, if introducing a new variable X_3 with coefficients $[c_3,a_{13},a_{23},a_{33}]=[2,1,1,1]$

2. (15 points) Consider the problem of assigning for four operators to four machines. The assignment costs (in NT dollars) are given at the table below. Operators 2 can not be assigned to machine 2. Also, operator 3 can not be assigned to machine 1. Suppose a fifth machine is made available. Its respective assignment costs (in NT dollars) to the four operators are 5k, 8k, 3k, and 4k, respectively. The new machine replaces an existing one if the replacement can be justified economically. (show all the optimal solutions)

(a) (8 points) Formulate the problem as an assignment model and find the optimal solution also calculate the cost or profit). In particular, is it economical to replace one of the existing machines? If so, which one? What will be the minimum costs?

(b) (7 points) Answer (a) when all the numbers are profits instead of costs.

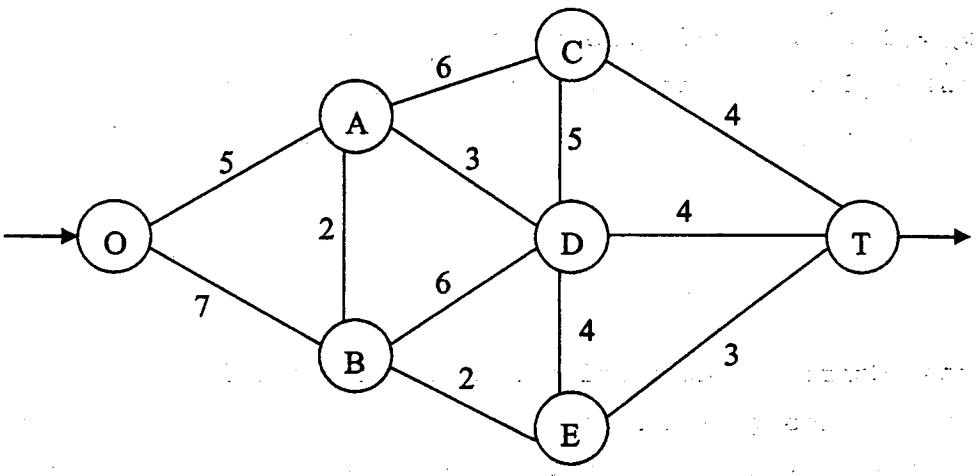
		Machine			
		1	2	3	4
Operator	1	7	3	6	7
	2	5	-	4	7
	3	-	6	5	4
	4	8	4	5	8

注意：背面尚有試題

3. (15 points) Solve the following shortest path problem.

(a) (10 points) Show all the optimal paths.

(b) (5 points)What is the minimum distance from O to T?



4. (20 points) Customers randomly arrive to a small hamburger stand with one server at the rate of 24 per hour. However, if there are n customers already there, a fraction n/3 of the arriving customers goes elsewhere. Customers are served first-come, first-serve according to an exponential distribution with a mean service time of 3 minutes.

(a) (2 points) What is the maximum number of customers in the system? (including those waiting on line and the one being served)

(b) (2 points) What is the percentage of no one in the system?

(c) (8 points) If each customer requires 10 square feet of counter space, what is the minimum counter space that must be provided to accommodate all customers at the stand at least 70 percent of the time?

(d) (8 points) On the average, how many customers are waiting on the line?

5. (20 points) Consider an electronic system consisting of three components, each of which must work for the system to function. The reliability of the system can be improved by installing several parallel units in one or more of the components. The following table gives the probability that the respective components will function if they consist of one, two, or three parallel units:

	Probability of Functioning		
Parallel Units	Component 1	Component 2	Component 3
1	0.5	0.6	0.5
2	0.6	0.7	0.7
3	0.8	0.8	0.9

The probability that the system will function is the product of the probabilities that the respective components will function.

The cost (in hundreds of dollars) of installing one, two, or three parallel units in the respective components is given by the following table:

	Cost		
Parallel Units	Component 1	Component 2	Component 3
1	2	2	2
2	3	4	3
3	4	5	4

Because of budget limitation, a maximum of $1,000 can be expended. Use dynamic programming to determine how many parallel units should be installed in each of the three components to maximize the probability that the system will function.