國立臺北科技大學

九十二學年度商業自動化與管理研究所入學考試

線性代數試題

填准考證號碼

注意事項：

1. 本試題共【八】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。
4. 作答時必須詳列計算或證明過程，配分列於各題。

1. Let $\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & -1 \\ 3 & 5 & 7 \end{bmatrix}$. Determine whether A is invertible and calculate A^{-1} if it is. (10%)

2. Let A be an $n \times n$ matrix. The adjoint of A is written $\text{adj}A$. If $\det A \neq 0$, then prove $A^{-1} = \frac{1}{\det A}\text{adj}A$. (10%)

3. Let $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$, find the eigenvalues of A, and the corresponding eigenvectors. (14%)

4. In P_3, determine whether the polynomials 1, x, x^2, and x^3 are linearly dependent or independent. (10%)

5. If A and B are similar $n \times n$ matrices, then prove A and B have the same eigenvalues. (10%)

6. Let T be a linear transformation from R^3 into R^2, and suppose that $T\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}$, $T\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\4\end{bmatrix}$, and $T\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}5\\-3\end{bmatrix}$. Compute $T\begin{bmatrix}3\\-4\\5\end{bmatrix}$. (10%)

7. Consider a square matrix of the form $M=\begin{bmatrix}A & 0\\P & B\end{bmatrix}$, where A is $p \times p$ and B is $q \times q$. Verify the following statements.

(a). If A is singular, so is M. (7%)
(b). If B is singular, so is M. (7%)
(c). If A and B are invertible, so is M. And, $M^{-1}=\begin{bmatrix}A^{-1} & 0\\-B^{-1}PA^{-1} & B^{-1}\end{bmatrix}$. (8%)

8. Construct an orthonormal basis in R^3 starting with the basis

$$\{v_1, v_2, v_3\}=\left\{\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix},\begin{bmatrix}1\\0\\1\end{bmatrix}\right\}. \text{ (14\%)}$$