

### 工程數學 (甲、乙、丁組) 試題

填准考證號碼

第一頁 共二頁

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#### 注意事項：

1. 本試題共  題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

一、A system consists of four switches is illustrated in Fig. 1.

- $A_1$  is the event that switch  $C_1$  is connected,
- $A_2$  is the event that switch  $C_2$  is connected,
- $A_3$  is the event that switch  $C_3$  is connected, and
- $A_4$  is the event that switch  $C_4$  is connected.

If  $A_1, A_2, A_3, A_4$  are independent events,  $P(A_1)=0.1, P(A_2)=0.2, P(A_3)=0.3,$  and  $P(A_4)=0.4,$  find the probability that electrical current can pass through the system. (5%)

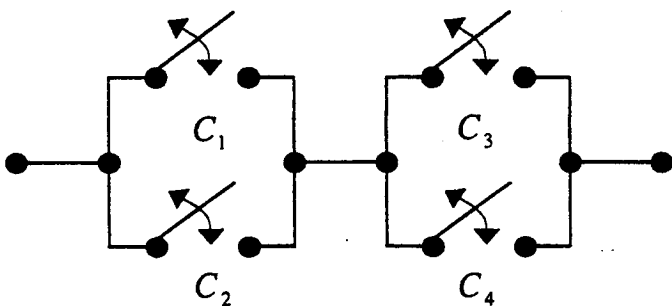


Figure 1

二、A continuous random variable  $X$  has a pdf (probability density function) of the form

$$f_X(x) = 2x/9 \text{ for } 0 < x < 3, \text{ and zero otherwise.}$$

- (a) Find  $E[X]$  (expected value of  $X$ ). (5%)
- (b) Find  $Var[X]$  (variance of  $X$ ). (5%)

(c) Find the probability  $P[|X - 1| \leq 1.5]$ . (5%)

三、 Let  $X$  and  $Y$  be continuous random variables with a joint pdf (probability density function) of the form  $f_{XY}(x, y) = k(x + y)$  for  $0 \leq x \leq y \leq 1$  and zero otherwise.

(a) Find  $k$  such that  $f_{XY}(x, y)$  is a joint pdf. (5%)

(b) Find the conditional pdf  $f_{Y|X}(y|x)$ . (5%)

(c) Find  $E[X^2Y]$ . (5%)

四、 A carton contains 144 baseballs, each of which has a mean weight of 5 ounces and a standard deviation of 0.4 ounces. Assume that the weights of individual baseballs are independent and let  $T$  represent the total weight of 144 baseballs in the carton.

(a) Use Chebychev inequality to bound the probability  $P(710 < T < 730)$ . (7%)

(b) Use central limit theorem to estimate the probability  $P(710 < T < 730)$ . (If you use any function in your answer, write the definition of the function used.) (8%)

五、 Let  $W$  be the subspace of  $R^4$  consisting of vectors of the form

$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ . All vectors  $\mathbf{x}$  in  $W$  satisfy  $x_1 + x_2 - x_3 = 0$  and  $x_2 - x_4 = 0$ .

(a) Find the dimension of  $W$ . (5%)

(b) Find a basis for  $W$ . (5%)

(c) Find an orthonormal basis for  $W$ . (5%)

六、 Let  $V$  be  $R^3$  and let  $S = \{X_1, X_2, X_3\}$  and  $T = \{Y_1, Y_2, Y_3\}$  be bases for  $R^3$ , where

$$X_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } Y_1 = \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix}, Y_2 = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}, Y_3 = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix}.$$

Compute the transition matrix  $P$  from the T-basis to the S-basis. (10%)

七、Let  $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ .

(a) Which matrix is diagonalizable? (5%)

(b) If  $A$  is diagonalizable, find a nonsingular matrix  $P$  such that  $P^{-1}AP$  is diagonal; if  $B$  is diagonalizable, find a nonsingular matrix  $Q$  such that  $Q^{-1}AQ$  is diagonal. (10%)

八、Find the rank of  $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 9 & -1 \\ -3 & 8 & 3 \\ -2 & 3 & 2 \end{bmatrix}$ . (10%)