

# 國立臺北科技大學

九十二學年度電機工程系碩士班入學考試

## 工程數學(丁、戊組)試題

填准考證號碼

第一頁 共一頁

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### 注意事項：

1. 本試題共【7】題，配分共100分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。

1. (10%) One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?
2. Consider two random variables X and Y. The conditional probability density function of X given Y has a Poisson distribution with rate y:

$$f_{X/Y}(x/y) = \frac{y^x e^{-y}}{x!} \quad x=0,1,2,\dots$$

The probability density function of variable Y is given by

$$f_Y(y) = \begin{cases} 2e^{-2y} & 0 < y < \infty \\ 0 & \text{else} \end{cases}$$

- a) (5%) Find joint probability density function of two random variables X and Y.
- b) (5%) Find the marginal probability density function of random variables X.
- c) (5%) Find the conditional probability density function of Y given X.
- d) (5%) Find the conditional expectation  $E[Y/X]$ .

(You may assume  $\int_0^{\infty} \mu^n e^{-\mu} d\mu = n!$  for a positive integer n)

3. (10%) Let X be a Gaussian random variable with probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

Find the probability density function of the random variable  $Z=4X^2$ .

4. (10%) Let  $X, Y$  be two independent random variables with probability density functions  $f_x$  and  $f_y$ , respectively. Let the random variable  $Z=X+Y$ , show that the probability density function of the random variable  $Z$  is  $f_z = f_x * f_y$  where  $*$  denotes the convolution operation.

5. (10%) Label the following statements as being true or false:(assume that all matrices in the following items are real, 2 points for each item with correct answer, -2 points for each item with incorrect answer, 0 points for each item with no answer)

- a) The rank of a matrix is equal to the number of its nonzero columns.
- b) The product of two matrices always has rank equal to the lesser of the ranks of two matrices.
- c) The determinant of a diagonal matrix is the product of its diagonal entries.
- d) If  $E$  is an elementary matrix, then  $\det(E)=\pm 1$ .
- e) Similar matrices always have the same eigenvectors.

6. Let  $D, E$  be ordered bases for  $\mathbb{R}_3$  given by

$$D = ([1, 2, 2], [3, 5, 1], [2, 1, 6])$$

$$E = ([1, 4, 1], [0, 4, 8], [0, 5, 1])$$

- a) (10%) Find the transition matrix from  $D$ -coordinates to  $E$ -coordinates.
- b) (5%) Let  $v=[3, 8, -1]$ , find  $[v]_D$  :the  $D$ -coordinates of vector  $v$ .

7. Consider the following matrix

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{bmatrix}$$

- a) (10%) Find the eigenvalues and the corresponding eigenvectors of  $A$ .
- b) (5%) Find the orthonormal matrix  $U$  such that  $A=UDU^T$  where  $D$  is diagonal.
- c) (5%) Give the definition of the Hermitian matrix. Is  $A$  a Hermitian matrix?
- d) (5%) Give the definition of the positive definite matrix. Is  $A$  a positive definite matrix?