國立臺北科技大學

九十二學年度資訊工程系碩士班入學考試

離散數學試題

填准考證號碼

注意事項：

1. 本試題共 [redacted] 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Represent the postfix expression as a binary tree and write the prefix form, the infix form, and the fully parenthesized index form of the expression (5%)

   ABC**CDE+/-

2. Rank the functions given below from smallest to largest complexity class. If some are in the same class, indicate so. (5%)

   $$2(\log n)^2, \frac{3n^2-5n}{2}, \sqrt{\log n}, 2^{\sqrt{n}}, (n+2)^2, \log n!, \frac{1}{n}, 2^{\sqrt{\log n}}$$

3. If the algorithm eliminates at least 1/6 of its problem size during each iteration, an iteration takes O(log n) time, and the algorithm terminates for $n \le 10$, what is the resulting time? (5%)

4. Simplify the expressions to equivalent statements that have as few symbols as possible.
   (1) $(p \wedge (p \vee q)) \vee q$ (2%)
   (2) $\neg(\neg(p \vee r) \vee \neg(p \vee q))$ (3%)

5. Let $A, B, C \subseteq U$. Prove that $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $C \subseteq A$. (10%)

6. Use mathematical induction to show that $\binom{m}{r+1} + \sum_{k=0}^{n-1} \binom{m+k}{r} = \binom{m+n}{r+1}$ for $n \ge 1$, $m, r \in Z^+$. (10%)

7. Let A and R be defined by: $A = \{0, 1, 2, 3, 4, 5\}$, $R = \{(x, y) \mid |x - y| = 0 \text{ or } |x - y| = 2\}$,
   (1) Draw $R \subseteq A \times A$ as a directed graph. (5%)
   (2) Is $R \subseteq A \times A$ reflexive? symmetric? transitive? Explain your answers. (10%)

8. (10%) One of twelve keys opens the door. If we try the keys one after another,
   (1) what is the probability that the door is opened on the first attempt?
   (2) On the second attempt?
   (3) On the twelfth attempt?

9. Consider that a server wants to pass a binary tree T as in Figure 1 to a client by sending a traversal of T.
   (1) Suppose the server sends the tree T in post-order CDBEA. Can the client re-construct the original tree T uniquely by following the received post-order traversal? In other words, whether the client can have only one tree representation for the post-order traversal sent by the server? Show your reason. (5%)
   (2) Suppose the server now sends T in preorder. Does the conclusion in (1) still hold? If you think the client can have only one tree representation for the preorder traversal sent by the server, please prove it. Otherwise, provide a modified preorder traversal which allows the client to re-construct the original binary tree uniquely (i.e., the tree representation for the modified preorder traversal is unique). (5%)

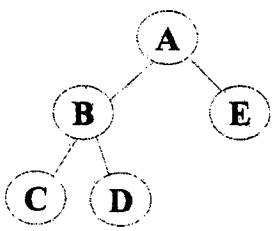

Figure 1: A binary tree T.

10. Let $(\mathbf{B}, \wedge, \vee, \neg, 1, 0)$ be a Boolean algebra. Define the operation $\oplus$ in the Boolean algebra as $p \oplus q = (p \vee q) \wedge \neg(p \wedge q)$.
   (1) Show that $p \oplus q$ is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$ (5%)
   (2) Does $(\mathbf{B}, \oplus)$ form an abelian group? Show your reason. (10%)

11. Consider the language

$$L = \{a^k b^k \mid k \geq 1\}$$

   (1) Give a grammar that specifies L. (5%)
   (2) Show that L is not a finite state language. (5%)

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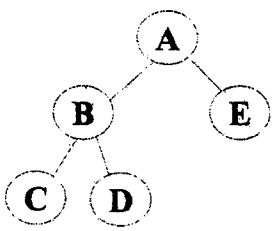

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