

國立臺北科技大學

九十二年學年度化學工程系碩士班入學考試

工程數學試題

填准考證號碼

第一頁 共一頁

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注意事項：

1. 本試題共【6】題，配分共100分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Schrödinger's Equation is given by

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r} \frac{d\Psi}{dr} + \frac{8\mu\pi^2}{h^2} \left(E + \frac{Ze^2}{r} \right) \Psi = 0$$

, where $E < 0$

(a) By means of the substitution $x = 2\lambda r$, where $\lambda = (\sqrt{8\pi^2\mu|E|})/h$, the above equation becomes

$$\frac{d^2\Psi}{dx^2} + \frac{2}{x} \frac{d\Psi}{dx} + \left(\frac{p}{x} - \frac{1}{4} \right) \Psi = 0. \quad (1)$$

Determine p . (5 points)

(b) By means of the substitution $\Psi(x) = e^{-\frac{x}{2}} u(x)$, Equation (1) becomes

$$x \frac{d^2u}{dx^2} + (2-x) \frac{du}{dx} + g(x)u(x) = 0 \quad (2)$$

Determine $g(x)$. (5 points)

(c) By differentiation of the following Laguerre's Differential Equation,

$$xy'' + (1-x)y' + py = 0$$

Equation (2) can be obtained with the identification of $u(x) = y'(x)$. Find a solution of Laguerre's Differential Equation. (10 points)

2 · Evaluate the inverse of the following Laplace Transform (20 points)

(a) $L^{-1}\left\{\frac{1}{\sqrt{S(S-1)}}\right\}$

(b) $L^{-1}\left\{\frac{\pi}{2} - \tan^{-1}\frac{S}{2}\right\}$

3 · Expand the function in a Fourier integral and determine what the integral representation converge to ? (15 points)

$$f(x)=x \text{ as } -\pi \leq x \leq \pi \quad \text{and} \quad f(x)=0 \quad |x| > \pi$$

4 · Solve the following partial differential equation (20 points)

$$\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} = 0 \quad (X > 0, Y > 0)$$

$$U(0, Y) = 0 \quad (Y > 0)$$

$$U(X, 0) = F(X) \quad (X > 0) \quad \text{where } F(X) = X \text{ as } 0 \leq X \leq 2$$

$$F(X) = 0 \text{ as } X > 2$$

5 · Evaluate $\iint_S F \cdot n dA$ (10 points)

$$F = \vec{i} + xy\vec{j} + yz\vec{k} \quad S: x^2 + y^2 \leq z, \quad y \geq 0, \quad z \leq 4$$

6 · Solve $y''' - 6y'' + 12y' - 8y = \sqrt{x}e^{2x}$ (15 points)