

# 國立臺北科技大學

九十二學年度土木與防災研究所入學考試

## 工程數學試題

填准考證號碼

第一頁 共二頁

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### 注意事項：

1. 本試題共【4】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

### Problem 1: (25 分)

The following is a 2-dimensional Laplace Equation problem.

1. Please derive the Laplace equation from the rectangular coordinate system to the polar coordinate system. (10 分)
2. Please solve the following boundary value problem in the polar coordinate system (i.e., find the steady-state temperature  $u(r, \theta)$  in the semicircular plate shown in Fig. 1). (15 分)

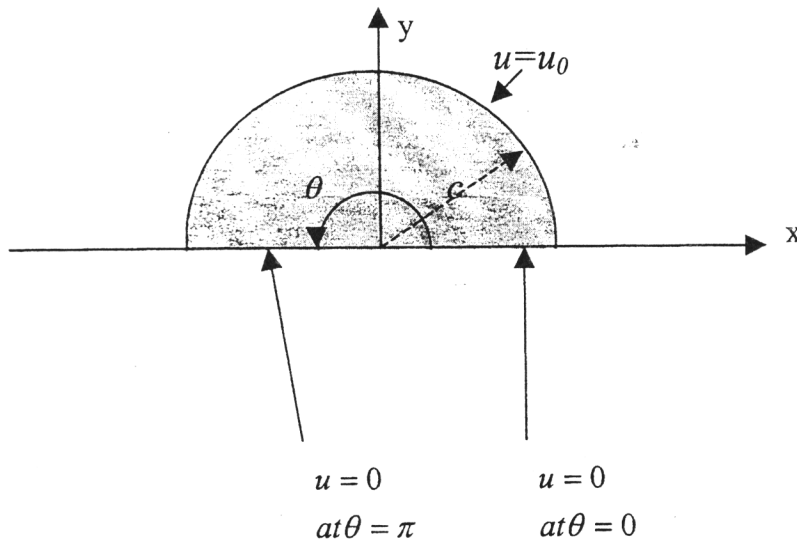


Fig. 1

**Problem 2: (25 分)**

Consider the spring/mass system of Fig. 2. Let  $x_1=x_2=0$  at the equilibrium position, where the weights are at rest. Choose the direction to the right as positive and suppose the weights are at positions  $x_1(t)$  and  $x_2(t)$  at time  $t$ . The equations of motion of the system are as shown as follows.

$$m_1 \ddot{x}_1 = -(k_1 + k_2)x_1 + k_2x_2 + f_1(t)$$

$$m_2 \ddot{x}_2 = k_2x_1 - (k_2 + k_3)x_2 + f_2(t)$$

These equations assume that damping is negligible but allow for forcing functions acting on each mass. Suppose  $m_1=m_2=1$  and  $k_1=k_3=4$  while  $k_2=2.5$ ; and suppose  $f_2(t)=0$  and  $f_1(t)=2[1-H(t-3)]$ . **Please use the Laplace transform to solve this system.**

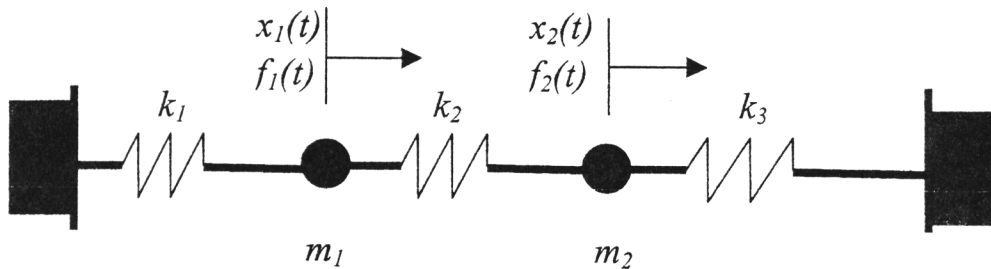


Fig. 2

**Problem 3: (25 分)**

$$\frac{dX(t)}{dt} = AX(t) + G(t) = \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix} X(t) + \begin{pmatrix} 8 \\ 4e^{3t} \end{pmatrix}, \text{ where } X(t) \text{ is a } 2 \times 1 \text{ matrix, and } A \text{ is } 2 \times 2$$

matrix.

- (a) Find the solution of the above equation. (15 分)
- (b) Find the matrix  $A^{10}$ . (10 分)

注意：背面尚有試題

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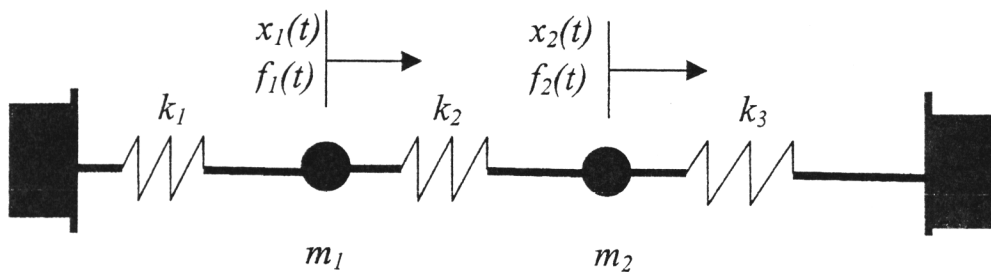


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**Problem 4: (25 分)**

The system of equations is shown as follow.

$$AX = B$$

$$A = \begin{pmatrix} 2 & 0 & 1 \\ -5 & 5 & 6 \\ -2 & 3 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

- Please use Gauss-Jordan elimination to solve the above equations. (15 分)
- Please use Cramer's rule to solve the above equations. (10 分)