國立臺北科技大學

九十二學年度光電技術研究所入學考試

工程數學試題

填准考證號碼 　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共【九】題，配分共100分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Find the general solution of the following equations

   (1) $y''+4y'+7y=13\cdot e^{-t}\cdot\sin(t+1)$. (10%)

   (2) $x^3y^{(3)}-2x^2y^{(2)}+13xy^{(1)}-13y=4-13\ln(x)$ (10%)

2. The Bernoulli equation is in the form of $P(x)\cdot y'+Q(x)\cdot y=R(x)\cdot y^{\alpha}$, in which α is a constant. Please find the integrating factor u(x,y). (10%)

   (Hint: try $u(x,y)=f(x)\cdot y^{-\alpha}$)

3. If $f(t+\omega)=f(t)$, so that $f(t)$ has a period ω. Prove that the Laplace transform of $f(t)$ is $L\{f(t)\}=\dfrac{1}{1-e^{-s\omega}}\displaystyle\int_0^{\omega}e^{-st}\cdot f(t)\cdot dt$. (10%)

4. Solve the initial value problem $y''+5\cdot y'+6\cdot y=f(t)$, $y(0)=y'(0)=0$, where

   $$f(t)=\begin{cases}-1, & 0\le t<2\\ 0, & t\ge 2\end{cases}$$ (10%)

5.

(1) Define $< f, g > = \int_0^{\pi} f(x) \cdot g(x) \cdot dx$, where *f*, *g* is in the vector space of functions continuous on [0, π]. Prove that *cos*(*x*), *cos*(2*x*),..., *cos*(n*x*) are mutual orthogonal. (7%)

(2) Compute the cosine series of the function $f(x) = 2 \cdot e^{-x}$, where $0 \le x \le 2$. (8%)

6. $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

(1) Find the eigenvalues and the corresponding eigenvectors of A. (4%)

(2) Find A^n for any positive integer n. (6%)

7. $A, P \in M_{n \times n}(C)$ and $P^{-1}AP$ is a diagonal matrix D.

Prove that $e^A = P \cdot e^D \cdot P^{-1}$. (10%)

8. Derive the following formula by using the residue theorem. (8%)

$$\int_{-\infty}^{\infty} \frac{dx}{\left(1 + x^2\right)^n} = \frac{\pi \cdot (2n - 2)!}{2^{2n-2} \cdot [(n-1)!]^2}$$

9. Evaluate $\oint_C \frac{[\cos(z) + \sin(z)]}{(z + i)^4} \cdot dz$, where C is any simple closed path about $-i$. (7%)