國立臺北科技大學

九十二學年度冷凍空調工程系碩士班入學考試

工程數學試題

填准考證號碼　　　　　　　　　　　　　　

注意事項：

1. 本試題共【五】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

(1) (20%) Solve the following ordinary differential equations.

(a) (5%) $y' = -\dfrac{2xy^3 + 2}{3x^2y^2 + 8e^{4y}}$

(b) (5%) $y'' + 4y = x + 2e^{-2x}$

(c) (5%) $(2x - 5y + 3)dx - (2x + 4y - 6)dy = 0$

(d) (5%) $x^2(x+1)y'' - x(2 + 4x + x^2)y' + (2 + 4x + x^2)y = -x^4 - 2x^3$ (***Hint***: $y = x$ is a homogeneous solution)

(2) (15%) Apply the Laplace transform to solve $y'' + 2ty' - 4y = 1$; $y(0) = y'(0) = 0$.

(3) (35%) Suppose a substance has density $\rho(x,y,z)$, specific heat $c(x,y,z)$, and thermal conductivity $k(x,y,z)$. Let $T(x,y,z,t)$ be the temperature of this substance at time t and point (x,y,z). Consider an imaginary smooth closed surface Σ within the substance, bounding a solid region M. If N is the unit outer normal to Σ, from the energy balance we have

$$\iint_{\Sigma} (k\nabla T) \bullet \mathrm{N}\, d\sigma = \iiint_M \rho c \frac{\partial T}{\partial t} dV .$$

(a) (5%) Briefly explain the Gauss's divergence theorem.

(b) (10%) Apply the Gauss's divergence theorem to the above energy balance result to obtain the heat conduction partial differential equation.

(c) (20%) If the thermal conductivity k is constant and the substance is one-dimensional with length l and has the following conditions, find the solution of $T(x,t)$.

$$\text{Boundary conditions } \begin{cases} T(0,t) = A \\ T(l,t) = B \end{cases},$$

$$\text{Initial condition } T(x,0) = \phi(x) \text{ , } 0 < x < l.$$

(4) (20%) $f(x) = 3x^2$ for $0 \le x < 4$ and $f(x+4) = f(x)$ for all x.

(a) (15%) Find the phase angle form of the Fourier series of the function.

(b) (5%) Plot some points of the amplitude spectrum of the function.

(5) (10%) Explain briefly the concepts and procedures to solve a system $\mathbf{X}' = \mathbf{AX}$ with $\mathbf{A}$ an $n \times n$ matrix of real numbers.