國立臺北科技大學 103 學年度碩士班招生考試

系所組別：2110、2120、2130 電機工程系碩士班甲、乙、丙組

第三節　工程數學　試題

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Consider the differential equation: $y' = \dfrac{x+2y+6}{-2x+y-2}$. Find the general solution. (15%)

2. Consider the differential equation: $(x^2+1)y'' - 2xy' + 2y = x(x^2+1)^2$ with the homogeneous solution $y = c_1 x + c_2(x^2-1)$. Find the particular solution. (20%)

3. Solve the differential equation: $y''(t) + 4y(t) = e^{-2t}\cos(2t) + \delta(t)$ with zero initial conditions. (15%)

4.

$$\text{Let } A = \begin{bmatrix} 1 & 1/3 & -2/3 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & -1/6 & 1/3 & 0 & 0 \\ -1/2 & -2/3 & 1/3 & 1/2 & 0 \\ 0 & -1/3 & 0 & 0 & 1/3 \end{bmatrix} \text{ and } b = \begin{bmatrix} 5 \\ 5 \\ 1 \\ 7 \\ 8 \end{bmatrix};$$

(a) Solve for x such that $Ax = b$; (5%)

(b) Find all eigenvalues of A, and list them in descending order; (5%)

(c) Find a 5×5 matrix X and X^{-1} such that $X^{-1}AX = D$, where D is the diagonal matrix whose diagonal elements are eigenvalues of A in descending order; (5%)

(d) Find the determinant of A and the inverse of A.(10%).

(e) Find e^A. (5%)

5. Let L be the linear mapping from R^3 to R^3 defined by $L(x) = Ax$, where

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ 7 & -6 & 3 \end{bmatrix}, \text{ and let } v_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \ v_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \ v_3 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}.$$

Find the transition matrix corresponding to a change of basis from $[e_1, e_2, e_3]$ to $[v_1, v_2, v_3]$, and use it to determine the matrix B representing L with respect to $[v_1, v_2, v_3]$. (10%)

6.

$$\text{Let } A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \\ 5 & -2 \\ -6 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 7 \\ -7 & 2 \\ 4 & 5 \\ 2 & -1 \end{bmatrix};$$

Find AB^T, A^TB and the inner product $\langle A, B \rangle$. (10%)