

# 國立臺北科技大學 102 學年度碩士班招生考試

系所組別：2220 電腦與通訊研究所乙組

## 第一節 工程數學 試題

第一頁 共一頁

### 注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、If  $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 1 \\ -\frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{2} \end{bmatrix}$

1. (10%) Evaluate the determinant of  $A$ .
2. (5%) Evaluate the determinant of  $2A^{-1}$ .

二、Consider the following linear operator  $T: R^3 \rightarrow R^3$ :

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix}$$

1. (5%) Find the matrix of  $T$  with related to the standard basis  $B = \{e_1, e_2, e_3\}$ .
2. (5%) Is it one-to-one? Please explain your answer.
3. (10%) Find a basis for  $R^3$  relative to which the matrix for  $T$  is diagonal.

三、If  $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}$

1. (5%) Find the eigenvalues of  $A$ .
2. (5%) Find  $A^7$ .
3. (5%) Find the trace of  $A^{12}$ .

四、Suppose events are occurring in time according to a Poisson distribution with a rate of  $\lambda t$  events per  $t$  hours. Let  $X$  denote the inter-arrival time between two events.

1. (7%) Find the pdf of  $X$ .
2. (3%) Does  $X$  satisfy memoryless property? Please explain your answer.

五、Suppose  $X$  and  $Y$  are continuous random variables.

1. If  $Y$  is selected at random from the interval  $(0,1)$ ,  $X$  is then selected at random from the interval  $(0,Y)$ . Please answer the following questions.
  - (a) (7%) Find the pdf of  $X$ .
  - (b) (3%) Are  $X$  and  $Y$  independent? Please explain your answer.
2. If  $(X,Y)$  is uniformly distributed on the region  $\{(x,y): 0 \leq x \leq y \leq 1\}$ . Please answer the following questions.
  - (a) (3%) Find the pdf of  $X$ .
  - (b) (3%) Compute the conditional density  $f_{Y|X}(y|x)$ .
  - (c) (4%) Are  $X$  and  $Y$  uncorrelated? Please explain your answer.

六、(10%) Suppose  $X$  and  $Y$  are independent random variables with the same probability of success  $p$  as

$$f_X(x) = p(1-p)^x, \quad f_Y(y) = p(1-p)^y, \quad \text{where } x, y \in \{0,1,2,\dots\}.$$

Let  $Z = \min(X, Y)$ . Find the pdf of  $Z$ .

七、(10%) The random variables  $X$  and  $Y$  have joint pdf

$$f_{X,Y}(x,y) = \frac{1}{2} \sin(x+y) \quad 0 \leq x \leq \frac{\pi}{2}, \quad 0 \leq y \leq \frac{\pi}{2}$$

Find the covariance of  $X$  and  $Y$ .