

國立臺北科技大學 103 學年度碩士班招生考試

系所組別：1111、1112、1120、1131、1132、1133

機電整合研究所甲、乙、丙組

第二節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共 4 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

Problem 1. (25%)

a)(10%) If $f(z) = u(x, y) + iv(x, y)$, where $z = x + iy$ is analytic function

$$\text{and } u(x, y) = x + y/(x^2 + y^2)$$

find $v(x, y) = ?$ and $f(z) = ?$

b)(15%) Evaluate the integral of

$$I = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx$$

Problem 2. (25%)

a)(10%) Please find the following integral

$$J(x) = \int_0^{\infty} e^{-t^2 - (x/t)^2} dt$$

b)(15%) Please find the solution of

$$(x^2 - 2x)y'' + 2(1 - x)y' + 2y = 0$$

Boundary condition

$$y(x = 0) = 1 \quad y(x = 1) = 1$$

Problem 3. (25%)

a)(10%) By using the periodic function of $f_1(x) = (\pi - x)/2$
in $x \in [-\pi, \pi]$, we try to find the solution of

$$f_2(x) = \sum_{n=1}^{\infty} \cos(nx) / n^2$$

Please find the $f_2(x) = ?$

b)(15%) By using the result of a) to find the summation result of

$$\sum_{n=1}^{\infty} 1/n^4 = ?$$

Problem 4 (25%)

Please solve the following partial differential equation

$$\frac{\partial^2 \psi}{\partial \xi^2} = \frac{\partial^2 \psi}{\partial \eta^2} - \sin^2 \eta \quad 0 \leq \xi < \infty \quad 0 \leq \eta < \infty$$

Boundary condition $\psi(\xi = 0, \eta) = 0$ $\psi(\xi = \infty, \eta)$ is bounded

Initial condition $\psi(\xi, \eta = 0) = 0$ $\frac{\partial \psi}{\partial \eta}(\xi, \eta = 0) = 0$