

國立臺北科技大學 102 學年度碩士班招生考試

系所組別：3510 化學工程研究所甲組

第三節 工程數學 試題

第一頁，共一頁

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Assume S is the surface of the hemispherical region bounded by a sphere

$$x^2 + y^2 + (z-1)^2 = 9 \text{ for } z \geq 1.$$

- (a) Find the unit outward normal vector and the tangent plane to the hemispherical surface at the point $(1, 2, 3)$. (10%)

- (b) If $\vec{F} = x\vec{i} + y\vec{j} + (z-1)\vec{k}$ and \vec{n} is the unit outward normal vector, evaluate

$$\iint_S (\vec{F} \cdot \vec{n}) dA. \quad (10\%)$$

2. Solve the differential equation $\frac{dy}{dx} + \cos^2(x-y+1) = 1$, $y(0) = 1$. (15%)

3. Find the general solution of $xy''(x) + (1-x)y'(x) - \frac{1}{x}y(x) = x$. (15%)

4. Let $X(s)$ and $Y(s)$ be the Laplace transforms of $x(t)$ and $y(t)$, respectively. If

$$X(s) = \frac{2s+1}{s(s^2+2s+e^{-s})} \text{ and } Y(s) = \frac{e^{-s}}{s(s^2+2s+e^{-s})}, \text{ evaluate } \int_0^{\infty} [x(t) - y(t)] dt. \quad (10\%)$$

5. Solve the following initial value problem for $y(t)$. (20%)

$$y''(t) + 4y'(t) + 3y(t) = f(t), \quad y(0) = 0, \quad y'(0) = 1, \text{ and } f(t) = \begin{cases} 2e^{-t}, & 0 \leq t < 2 \\ 3e^{-t}, & t \geq 2 \end{cases}.$$

6. Solve the following partial differential equation by the Fourier transform. (20%)

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} - 3t^2 y \quad (t \geq 0, 0 \leq x < \infty)$$

$$y(x, 0) = e^{-x} \quad (0 \leq x < \infty)$$

$$\frac{\partial y}{\partial x}(0, t) = 0 \quad (t \geq 0)$$