

國立臺北科技大學 102 學年度碩士班招生考試

系所組別：2110、2120、2130 電機工程系碩士班

甲、乙、丙組

第二節 工程數學 試題

第一頁，共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) Solve $2(y^3 + x^2y + x)dy + (2x^3y^2 + 4x^2y + 2xy^2 + xy^4 + 2y)dx = 0$
(Hint: find an integrating factor)

2. (15%) Solve $x^2y'' - xy' + 4y = \cos(\ln(x)) + x \sin(\ln(x))$

3. (20%) Solve $\begin{cases} x'' + y' + 3x = 15e^{-t} \\ y'' - 4x' + 3y = 15\sin(2t) \end{cases}$
subject to $x(0) = 35, x'(0) = -48, y(0) = 27, y'(0) = -55$

4. Given the vectors $\mathbf{y} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}, \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Let W be the subspace spanned
by $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

(10%) (a) Represent \mathbf{y} as the sum of a vector \mathbf{y}_1 in W and a vector \mathbf{y}_2 in the orthogonal complement of W .

(5%) (b) Determine whether the matrix $A = [\mathbf{y} \ \mathbf{u}_1 \ \mathbf{u}_2 \ \mathbf{u}_3]$ is invertible, and justify your answer.

5. Let $B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

(5%) (a) Find a basis for the column space of B .

(15%) (b) Find an orthogonal basis for the column space of B .

(5%) (c) Determine whether the number 0 is an eigenvalue of B , and justify your answer.

6. (10%) Determine the dimensions of the eigenspaces corresponding to the eigenvalues of the

matrix $C = \begin{bmatrix} 1 & -2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -2 & 1 \end{bmatrix}$.