

國立臺北科技大學 103 學年度碩士班招生考試

系所組別：2210 電子工程系碩士班甲組

第二節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共九題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、

Determine whether the set S below is a spanning set for R^3 . Justify your answer. (5%)
 $S = \{(1,1,1)^T, (1,1,0)^T, (1,0,0)^T\}$

二、

Determine $N(A)$, i.e., the Nullspace of matrix A , if $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$. (5%)

三、

Determine the least squares solution to $Ax = b$, where $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$. (10%)

四、

The matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 1 & 2 \\ -1 & 0 & 2 \end{bmatrix}$ has a LU -factorization, i.e., $A = LU$. Find the matrix L and U ,

where L is a lower triangular matrix with its diagonal entries equal to 1, and U is an upper triangular matrix. (15%)

五、

Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

- (a). Find the eigenvalues and the corresponding eigenvectors of matrix A . (10%)
- (b). Is matrix A diagonalizable? That is, can we find a nonsingular matrix S and a diagonal matrix D such that $S^{-1}AS = D$? If the answer is "Yes", find the resulted diagonal matrix D and the nonsingular matrix S that diagonalizes A . On the other hand, give the reason if your answer is "No". (10%)

六、

Let X be a uniformly distributed continuous random variable with $E[X]=1$ and $E[X^2]=\frac{4}{3}$.

What is the probability density function of X ? (10%)

七、

Determine the probability of obtaining at least two "Six" in throwing a fair die 3 times. (10%)

八、

Determine the joint probability function $f(X, Y)$ in the game of throwing a fair die 3 times, where X is the number of 1s and Y is the number of 6s. (10%)

九、

X is a discrete random variable which assumes -1, 0, 1 with probability $1/3$, and $Y = X^2$.

- (a). Determine the mean and variance of Y . (6%)
- (b). Determine the covariance σ_{XY} of X and Y . (6%)
- (c). Determine the correlation coefficient ρ of X and Y . (3%)