國立臺北科技大學 103 學年度碩士班招生考試

系所組別：2401、2402、2403 光電工程系碩士班

第二節　工程數學　試題

注意事項：

1. 本試題共七題，配分共 100 分。

2. 請標明大題、子題編號作答，不必抄題。

3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

(1) (15%) Let $\sigma = \begin{pmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ be a 3×3 matrices. Please compute $\exp(i\sigma)$.

(2) (15%) The Laplace transform of a function $f(t)$ is defined to be $\mathcal{L}(f(t)) = F(s) = \int_{0^-}^{\infty} e^{-st} f(t)\, dt$. Please compute the Laplace transform of $f(t) = \exp(\omega t)$.

(3) (15%) Please use Laplace transform to solve the following initial value problem.
$$y''(t) + 3y'(t) + 2y(t) = 0 \ , \ y(0) = 1 \ , \ y'(0) = 0 \ .$$

(4) (15%) Use the residue theorem to evaluate the following integral.
$$\int_{-\infty}^{\infty} \frac{e^{iz}}{z^2 + 5^2} dz \ .$$

(5) (10%) Let (r, θ) be the polar coordinate system on (x, y) plane. The region D is given by $0 \leq r \leq 2$ and $0 \leq \theta \leq \pi$. Please evaluate
$$\int\int_D \sin((r\cos(\theta))^2 + (r\sin(\theta))^2)\, dA \ ,$$
where dA is the area element.

(6) (15%) Solve the following differential equation by the method of separations of variables.
$$y\frac{dy}{dx} = \sin(x)e^{-y} \ .$$

(7) (15%) Solve the following ordinary differential equation for $y(x)$,
$$\frac{d^2y}{dx^2} + 4y = 5e^{-x} \ .$$