

國立臺北科技大學 101 學年度碩士班招生考試

系所組別：2220 電腦與通訊研究所乙組

第一節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、

If $\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 3 & 4 & 3 & 0 & 0 \\ 1 & 5 & 1 & 4 & 0 \\ 2 & 4 & 1 & 6 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 2 & 1 & 2 & 1 \\ 0 & 2 & 3 & 3 & 6 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$, find the determinant of \mathbf{AB} .

(10%)

二、
Find the eigenvalues of the following matrices:

1. $\mathbf{A} = \begin{bmatrix} 4 & 3 & 0 & 0 & 0 \\ -2 & -1 & 0 & 0 & 0 \\ 1 & 7 & 10 & 1 & -7 \\ 2 & -1 & 0 & 5 & 0 \\ -3 & 1 & 6 & -4 & -3 \end{bmatrix}$ (10%)

2. $\mathbf{B} = \begin{bmatrix} 2 & 2 & \cdots & 2 \\ 2 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \cdots & 2 \end{bmatrix}_{100 \times 100}$ (10%)

三、

Find the QR decomposition of matrix $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. (20%)

四、

If the joint probability density function (p.d.f.) of two random variables X and Y is $f_{X,Y}(x,y) = 6e^{-(3x+2y)}$, $x \geq 0, y \geq 0$, what is their correlation coefficient? (10%)

五、

Suppose X and Y are independent Normal variables with zero mean and common variance σ^2 . Define $Z = (X^2 + Y^2)^{1/2}$, and $W = \tan^{-1}(Y/X)$, where $|W| < \pi$. Find the joint p.d.f. of Z and W . (15%)

六、

Assume that X and Y are two random variables, satisfying $Y = X^2 + 1$. Find $P(Y < 4)$, given that X is a Binomial random variable with the p.d.f. $P(X = x) = \binom{3}{x} p^x (1-p)^{3-x}$, where $x = 0, 1, 2, 3$, and $P(X \geq 1) = 26/27$. (15%)

七、

There are two random variables X and $Y \in \{0, 1\}$. If $P(X=0, Y=0) = P(X=0, Y=1) = P(X=1, Y=0) = 1/4$, what is the covariance between $(3X+2)$ and $(4Y+9)$? (10%)