

國立臺北科技大學 101 學年度碩士班招生考試

系所組別：1521、1522、1523 自動化科技研究所 乙組

第一節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) For vector space

(1)(5%) Let v_1, v_2, \dots, v_m are the vectors of vector space V . Explain that linearly dependent and linearly independent for v_1, v_2, \dots, v_m .

(2)(5%) Are $\cos(x)$ and $\sin(x)$ linearly dependent or linearly independent? Why?

(3)(5%) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$, $D = \begin{bmatrix} -1 & -2 \\ 0 & -2 \end{bmatrix}$. Are A, B, C, D, linearly dependent or linearly independent? Why?

2. (20%) If u and v are any two vectors in an inner product space V , then

$$|\langle u, v \rangle| \leq \|u\| \|v\|$$

Equality holds if and only if u and v are linearly dependent. Please prove this Cauchy-Schwarz theorem.

3. (20%) For each matrix, find the characteristic equation, and the eigenvalues and associated eigenvectors

(1)(10%)

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

(2)(10%)

$$\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$$

4. (15%) Find an orthonormal basis for this subspace of \mathbb{R}^4 .

$$\left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \mid x - y - z + w = 0 \text{ and } x + z = 0 \right\}$$

5. (15%) Show that this gives the equation of a line in \mathbb{R}^2 thru (x_2, y_2) and (x_3, y_3) .

$$\begin{vmatrix} x & x_2 & x_3 \\ y & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

6. (15%) Solve the differential equations

$$mx_1'' + (k + k_1)x_1 - k_1x_2 = 0$$

$$mx_2'' + (k + k_1)x_2 - k_1x_1 = 0$$

where m , k , and k_1 are positive real number.