注意事項：

1.本試題共六題，配分共 100 分。

2.請標明大題、子題編號作答，不必抄題。

3.全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Find the general solution of the differential equation (15 points).

$$y'' - y = \frac{2}{1+e^x}.$$

2. Compute the inverse Laplace transform (15 points).

$$\mathcal{L}^{-1}\left[\frac{se^{-3s}}{s^2+4}\right].$$

3. Let $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Compute A^{100}. (15 points).

4. Evaluate the line integral $\int_c (5-xy-y^2)dx - (2xy-x^2)dy$, where C is the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$, traversed counterclockwise. (15 points).

5. Let f be a scalar field and $\mathbf{F}$ a vector field. State whether each expression is not meaningful, a scalar field or a vector field. (20 points).

(a) curl f

(b) grad f

(c) curl(grad f)

(d) grad $\mathbf{F}$

(e) div(grad f)

(f) grad(div $\mathbf{F}$)

(g) div (div $\mathbf{F}$)

(h) div $\mathbf{F}$

(i) curl(curl $\mathbf{F}$)

(j) div(curl(grad f))

6. Determine that solution of the partial differential equation

$$4\frac{\partial f(x,y)}{\partial x} + 3\frac{\partial f(x,y)}{\partial y} = 0$$

which satisfies the condition $f(x,0) = \sin x$ for all x. (20 points).