

國立臺北科技大學 102 學年度碩士班招生考試

系所組別：2401、2402、2403、2404 光電工程系碩士班

第一節 工程數學 試題

第一頁，共一頁

注意事項：

1. 本試題共 10 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

- (1) (10%) Solve the following ordinary differential equation for $y(x)$ by the method of separations of variables.

$$(x^3 + 7) \frac{dy}{dx} = x^2 y$$

- (2) (10%) Solve the following ordinary differential equation for $y(x)$. Please write down the general solution.

$$\frac{d^2 y}{dx^2} + y = 3x^2 e^{-x}$$

- (3) (10%) Let $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ be two 2×2 matrices. Please compute $\exp(i(\sigma_1 + \sigma_2))$.

- (4) (10%) The Laplace transform of a function $f(t)$ is defined to be $\mathcal{L}(f(t)) = F(s) = \int_0^\infty e^{-st} f(t) dt$. Please show that the Laplace transform of the n -th derivative of $f(t)$ is given by

$$\mathcal{L}(f^n(t)) = s^n F(s) - s^{n-1} f(0) - s^{n-2} f^1(0) - \dots - f^{(n-1)}(0).$$

- (5) (10%) Please use Laplace transform to solve the following initial value problem.

$$y'(t) + y(t) = u(t), \quad y(0) = 0, \quad u(t) \text{ is the unit step function.}$$

- (6) (10%) Please find the Fourier transform of the Gaussian distribution $f(x) = \exp(-\frac{x^2}{2a})$.

- (7) (10%) Use the residue theorem to evaluate the following integral.

$$\int_C \frac{dx (1+x)^{50}}{x^{10}},$$

where C is a counterclockwise circle around origin with radius δ .

- (8) (10%) Let $\vec{F} = xy^2\mathbf{i} + yz^2\mathbf{j} + zx^2\mathbf{k}$ be a vector field. Compute the flux across the surface bounding the region defined by $3 \leq x^2 + y^2 + z^2 \leq 7$.
- (9) (10%) The Gamma function is given by $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$. Show that $\Gamma(n+1) = n!$ when n is a positive integer.
- (10) (10%) Please evaluate

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \exp(-(x^2 + y^2)) dy dx.$$