國立臺北科技大學103學年度碩士班招生考試

系所組別：2152 電機工程系碩士班戊組

第三節　離散數學　試題（選考）

注意事項：

1. 本試題共12題，配分共100分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (10%) Use mathematical induction to prove that

(a) (5%) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.

(b) (5%) $\sum_{i=1}^{n} f_{2i-1} = f_{2n}$, where f_i is the ith Fibonacci number (f_0=0, f_1=1, and f_i=f_{i-1}+f_{i-2}, $i > 1$).

2. (10%) Suppose that X and Y are events such that $p(X)$=0.5 and $p(X \cup Y)$=0.7. Find $p(Y)$ in each of the following cases:

(a) (2%) X and Y are disjoint.

(b) (4%) X and Y are independent.

(c) (4%) $p(Y|X)$=0.3.

3. (10%) Let $<_x$ be a relation on $\mathbb{Z}\times\mathbb{Z}$ such that $(a,b) <_x (c,d)$ if and only if $a \le c$ and $b \le d$, where $a, b, c, d \in \mathbb{Z}$ and $\mathbb{Z}$ is the set of integer numbers.

(a) (3%) Prove that $<_x$ is a partial order.

(b) (3%) Draw the Hasse diagram of the relation $<_x$ for $\mathbb{Z} = \{1, 2, 3\}$.

(c) (4%) How many maximal chains does the Hass diagram of part (b) have? (A maximal chain is one that is not a subset of another chain.)

4. (10%) Use Huffman coding to encode the following symbols with the probabilities of occurrence listed: a: 0.45, b:0.13, c:0.12, d:0.16, e:0.09, f:0.05.

(a) (5%) Draw the corresponding Huffman tree.

(b) (2%) List the code for each symbol.

(c) (3%) What is the average number of bits used to encode a symbol?

5. (10%) Solve the following recurrence relations.

(a) (5%) $a_n=6a_{n-1}-8a_{n-2}$, $n \geq 2$, $a_0=4$, $a_1=14$.

(b) (5%) $a_n=\frac{2}{3}a_{n-1}-\frac{1}{9}a_{n-2}$, $n \geq 2$, $a_0=1$, $a_1=1$.

6. (10%) For a given alphabet $\Sigma=\{0,1\}$, let L be the language consisting of all strings which have an even number of 1's.

(a) (5%) Draw a state diagram for a deterministic finite automata that recognizes the language L.

(b) (5%) Give a regular expression that specifies the language L.

7. (10%) Give a tight upper bound for each following function using Big-O notation.

(a) (2%) $f(n)=5000n\log n + n^2$

(b) (2%) $f(n)=10^{10}+10!$

(c) (2%) $f(n)=\sum_{i=1}^{n} 2^{10}$

(d) (2%) $f(n)=\sum_{i=1}^{n}\sum_{j=1}^{i} 2$

(e) (2%) $f(n)=\sum_{i=1}^{n}\sum_{j=1}^{n} n$

8. (10%) For each of the following determine whether $\otimes$ is an associative operation. Explain your answer. Suppose that $\mathbb{R}$ is the set of real numbers.

(a) (2%) Define $\otimes$ on $\mathbb{R}$ by letting $a\otimes b=ab+1$.

(b) (2%) Define $\otimes$ on $\mathbb{R}$ by letting $a\otimes b=a+b-1$.

(c) (2%) Define $\otimes$ on $\mathbb{R}$ by letting $a\otimes b=a^b$.

(d) (2%) Define $\otimes$ on $\mathbb{R}$ by letting $a\otimes b=\frac{a}{b}$.

(e) (2%) Define $\otimes$ on $\mathbb{R}^2$ by letting $(a,b)\otimes(c,d)=(ac, bc+d)$.

9. (10%) For each of the following, determine with proof whether or not it is a group.

(a) (5%) The set $\{1, -1, \sqrt{-1}, -\sqrt{-1}\}$ under multiplication.

(b) (5%) The set $\{3n \mid n\in\mathbb{Z}\}$ under addition, where $\mathbb{Z}$ is the set of integer numbers.

注意：背面尚有試題

10. (2%) Which one of the following propositions is a tautology?

(1) $(p\rightarrow q)\wedge(\neg p\rightarrow\neg q)$　(2) $(p\vee q)\rightarrow(\neg p\vee\neg q)$　(3) $(p\wedge\neg q)\vee(\neg p\wedge\neg q)$　(4) $(q\wedge(p\rightarrow\neg q))\rightarrow\neg p$

11.(2%) Which one of the following propositions logically implies all the others?

(1) $p\vee q$　(2) $p\rightarrow q$　(3) $q\wedge(p\rightarrow\neg q)$　(4) $(p\wedge q)\rightarrow(p\vee q)$

12. (6%) Determine whether each of the following statements is true or false. If false, explain why or provide a counterexample. If true, explain why. Suppose that the universe of discourse of each variable is the set of integer numbers.

(a) (2%) $\exists x\forall y(x^2=y)$

(b) (2%) $\exists x\forall y(x=y^2)$

(c) (2%) $\exists x\exists y\forall z(z=2x+2y)$