

國立臺北科技大學 102 學年度碩士班招生考試

系所組別：2140、2150 電機工程系碩士班丁、戊組

第二節 工程數學 試題

第一頁，共一頁

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Assume the cumulative distribution function (CDF) of a random variable T is given by

$$F_T(t) = \begin{cases} 1 - e^{-t/3}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(10%) (a) Find the probability density function (PDF) of the random variable T

(5%) (b) Find $P[2 \leq T < 4]$

(5%) (c) Find $E[T]$

2. Let U be a uniform (0,1) random variable.

(10%) (a) Assume $g(x)$ be an arbitrary function with an inverse $g^{-1}(u)$

defined for $0 < u < 1$. Let random variable $X = g^{-1}(U)$. Show that the cumulative distribution function (CDF) of X is $F_X(x) = g(x)$.

(10%) (b) Derive $f(U)$ such that $X = f(U)$ is the exponential (1) random variable.

3. (10%) A modem transmits one million bits. Each bit is 0 or 1 independently with equal probability. Let X_i be the value of bit i (either 0 or 1). The number of ones in one million bits is $W = \sum_{i=0}^{10^6} X_i$. Use the central limit theorem to approximate the probability of at least 502,000 ones in the transmission, i.e., $P[W \geq 502000] = ?$

[Express your answer using the standard normal CDF $\Phi(\cdot)$]

4. Given the vectors $\mathbf{y} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. Let W be the subspace

spanned by $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

(10%) (a) Represent \mathbf{y} as the sum of a vector \mathbf{y}_1 in W and a vector \mathbf{y}_2 in the orthogonal complement of W .

(5%) (b) Determine whether the matrix $A = [\mathbf{y} \ \mathbf{u}_1 \ \mathbf{u}_2 \ \mathbf{u}_3]$ is invertible, and justify your answer.

5. Let $B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

(5%) (a) Find a basis for the column space of B .

(15%) (b) Find an orthogonal basis for the column space of B .

(5%) (c) Determine whether the number 0 is an eigenvalue of B , and justify your answer.

6. (10%) Determine the dimensions of the eigenspaces corresponding to the eigenvalues of

the matrix $C = \begin{bmatrix} 1 & -2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -2 & 1 \end{bmatrix}$.