

103 學年度四年制二、三年級轉學生招生考試

四技二年級 光電工程系

第一節 微積分 試題

第一頁 共一頁

注意事項：

1. 本試題共 8 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

(1) (10 points) Given $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{px + q} - 3} = 9$, please find p and q .

(2) (10 points) Evaluate the integral

$$\int \frac{1}{x^4 + 4} dx.$$

(3) (15 points) Find the anti-derivative $\int \sec^{-1} x dx$.

(4) (15 points) Evaluate the integral $\int \tan^5 x dx$.

(5) (a) (10%) Please show that

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}, \quad \text{for } a > 0.$$

(b) (10%) Use the above result to compute

$$\int_{-\infty}^{\infty} x^8 e^{-ax^2} dx, \quad \text{for } a > 0.$$

(6) (10 points) Find the minimal surface area of a cylindrical can (with both bottom and top) with a fixed volume V .

(7) (a) (5 points) Compute

$$\lim_{x \rightarrow \infty} (3^x + 4^x + 5^x)^{\frac{1}{x}} .$$

(b) (5 points) Compute

$$\lim_{x \rightarrow 0} \frac{(\sin^{-1} 4x)^2}{x \tan^{-1} 7x} .$$

(8) (10 points) Solve the first order differential equation

$$x \frac{dy}{dx} = x^2 + 3y .$$