國立臺北科技大學 102 學年度碩士班招生考試

系所組別：2210 電腦與通訊研究所甲組

第一節 工程數學 試題

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Two square matrices P and Q are given by

$$P=\begin{bmatrix}1 & 1 & 1 & 1\\ 2 & 4 & 6 & x\\ 4 & 16 & 36 & x^2\\ 8 & 64 & 216 & x^3\end{bmatrix},\qquad Q=\begin{bmatrix}1 & 0 & 0\\ 0 & \sin\phi & -\cos\phi\\ 0 & \cos\phi & \sin\phi\end{bmatrix}.$$

(a) If the determinant of P equals 240, find the value of x, being a positive integer. (6%)

(b) Find the inverse matrix of Q, Q^{-1}. (6%)

2. Let V and U be vector spaces, and let E and F be the ordered basis for V and U, respectively.

$$E=[\mathrm{v}_1,\ \mathrm{v}_2,\ \mathrm{v}_3]=[(1,\ 1,\ 1)^T,\ (1,\ 3,\ 2)^T,\ (1,\ 0,\ 4)^T]$$
$$F=[\mathrm{u}_1,\ \mathrm{u}_2,\ \mathrm{u}_3]=[(1,\ 1,\ 0)^T,\ (2,\ 3,\ 0)^T,\ (2,\ 3,\ 1)^T]$$

(a) Find the transition matrix from E to F. (6%)

(b) Let $\mathbf{x}=3\mathrm{v}_1+\mathrm{v}_2-2\mathrm{v}_3$, find the coordinate of $\mathbf{x}$ with respect to the F. (5%)

(c) Verify your answer on (b). (5%)

3. Assume that the total population of certain metropolitan area remains relatively fixed. However, each year 10% of people living in the city move to the suburbs and 20% of the people living in the suburbs move to the city. If initially, 40,000 persons live in the city, and 50,000 persons live in the suburbs, that is, the initial vector $\mathbf{x}_0=[40000,\ 50000]^{\mathrm{T}}$.

(a) Derive the vector $\mathbf{x}_n$, $n\geq 0$. (10%)

(b) For the second year, find the vector $\mathbf{x}_2$. (6%)

(c) Find the limit vector $\mathbf{x}_k$, also called a steady-state vector. (6%)

4. Let ${}_n\mathrm{C}_k$ denote k-combinations of n objects, and ${}_n\mathrm{P}_k$ denote k-permutations of n distinguishable objects.

(a) Examine whether ${}_{2n}\mathrm{C}_2=4\cdot{}_n\mathrm{C}_2+2n$ is true or not? (6%)

(b) Examine whether ${}_n\mathrm{P}_k=k\cdot({}_{n-1}\mathrm{P}_{k-1})+{}_{n-1}\mathrm{P}_k$ is true or not? (6%)

5. Suppose that X and Y are continuous random variables with the joint PDF (probability density function) as follows:

$$f_{X,Y}(x,y)=\begin{cases}\dfrac{ky}{8}, & -1\leq x\leq 1,\ 0\leq y\leq(1-x^2).\\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the value of k. (10%)

(b) Find the marginal PDF of Y, $f_Y(y)$. (6%)

(c) Calculate the probability based on $\int_{0.5}^{1} f_Y(y)\,dy$. (6%)

6. A smart mobile phone transmits one packet in every 15-ms time slot over a Wi-Fi wireless LAN (WLAN). With probability p = 0.10, a packet is received in error, independent of any other packet. To save power energy when the link quality of WLAN is poor, the transmitter enters a timeout state whenever *four* consecutive packets are received in error. During a timeout, the smart phone performs an independent Bernoulli trial with success probability q = 0.125 in every slot. When a success occurs, the smart phone starts sending packets in the next slot as though no packets had been in error. Try to construct a Markov chain for this system, and specify the limiting state probabilities. (8%, 8%)