

102 學年度四年制二、三年級轉學生招生考試

四技二年級 光電工程系

第一節 微積分 試題

第一頁 共一頁

注意事項：

1. 本試題共 8 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

(1) (10%) Please evaluate

$$\int_{-7}^7 \int_0^{\sqrt{49-x^2}} \exp(-(x^2 + y^2)) dy dx.$$

(2) (a) (10%) Please show that

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}, \text{ for } a > 0.$$

(b) (10%) Use the above result to compute

$$\int_{-\infty}^{\infty} x^4 e^{-ax^2} dx, \text{ for } a > 0.$$

(3) (10%) Let

$$\vec{r}(t) = t^2 \vec{i} + \sin(t) \vec{j} + \cos^2(t) \vec{k}$$

be a parametric curve, where \vec{i}, \vec{j} , and \vec{k} are unit vectors in x, y , and z directions respectively. Compute the unit tangent vector to the curve at $t = \pi/4$.

(4) (a) (7%) Compute

$$\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right).$$

(b) (8%) Compute

$$\lim_{x \rightarrow 0} \frac{\sin^3 4x}{x^2 \tan x}.$$

(5) (15%) Let

$$F(x, y, z) = x^2 + \int_{\sin z}^{y^2} \sqrt{1+t^2} dt.$$

Find the tangent plane to $F(x, y, z) = 9$ at $(x, y, z) = (3, 0, 0)$.

(6) (10%) Evaluate

$$\int_0^{\ln 4} \frac{e^{2x}}{(e^x + 2)^{1/3}} dx.$$

(7) (10%) Given $g(2) = 5$, $f(2) = 3$, and $g'(x) = \sqrt{x^3 + 3}$, $f'(x) = \sqrt{x^4 + 3}$ for all $x > 0$, find the derivative of $g(f(x))$ at $x = 2$.

(8) (10%) Find the maximal volume of a cylindrical can (with bottom only and without top) with a fixed surface area A .