國立臺北科技大學九十八學年度碩士班招生考試

系所組別：1521 1522 1523 自動化科技研究所乙組

第一節　工程數學　試題

注意事項：

1. 本試題共四題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 作答須詳列計算過程，並將答案清楚標示，否則不予計分。

1. (30%) Consider the system $x'=\mathbf{A}x$ where $\mathbf{A}=\begin{bmatrix}7 & -1\\ 9 & 1\end{bmatrix}$

   (a) (10%) Find a vector v_1 such that $x_1(t)=e^{4t}v_1$ is a solution to the system.

   (b) (10%) Find two vectors u and w such that $x_2(t)=e^{4t}u+te^{4t}w$ is a second, linearly independent solution to the system.

   (c) (10%) Write the general solution to the system.

2. (20%) For what range of numbers a and b are the matrices **A** and **B** positive definite?

$$A=\begin{bmatrix}a & 2 & 2\\ 2 & a & 2\\ 2 & 2 & a\end{bmatrix} \qquad B=\begin{bmatrix}1 & 2 & 4\\ 2 & b & 8\\ 4 & 8 & 7\end{bmatrix}$$

3. (10%) Consider the system

$$\begin{aligned}2x_1-3x_2+5x_3&=0\\ -x_1+7x_2-x_3&=0\\ 4x_1-11x_2+kx_3&=0\end{aligned}$$

   For what value of k will the system have nontrivial solutions?

4. (40%) Construct a matrix with the required property or say why that is impossible.

   (a) (10%) Column space contains $\begin{bmatrix}1\\2\\-3\end{bmatrix}$ and $\begin{bmatrix}2\\-3\\5\end{bmatrix}$, nullspace contains $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

   (b) (10%) Row space contains $\begin{bmatrix}1\\2\\-3\end{bmatrix}$ and $\begin{bmatrix}2\\-3\\5\end{bmatrix}$, nullspace contains $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

   (c) (10%) $Ax=\begin{bmatrix}1\\1\\1\end{bmatrix}$ has a solution and $A^{\mathrm{T}}\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

   (d) (10%) The columns add up to a column of 0s, the rows add to a row of 1s.