

國立臺北科技大學九十八學年度碩士班招生考試

系所組別：1511 1512 自動化科技研究所甲組

第一節 工程數學 試題

第一頁 共一頁

**注意事項：**

1. 本試題共四題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 作答須詳列計算過程，並將答案清楚標示，否則不予計分。

1. (20%) Find the general solution to

(a) (10%)  $x^2y'' - 3xy' + 4y = 0, \quad x > 0.$

(b) (10%)  $x^2y'' + 5xy' + 13y = 0, \quad x > 0.$

2. (10%) Find all solutions to the nonhomogeneous system

$$\begin{cases} x' = 2x + y + t \\ y' = x + 2y + t^2 \end{cases}$$

3. (30%) Consider the system  $x' = Ax$  where  $A = \begin{bmatrix} 7 & -1 \\ 9 & 1 \end{bmatrix}$ .

(a) (10%) Find a vector  $v_1$  such that  $x_1(t) = e^{4t}v_1$  is a solution to the system.

(b) (10%) Find two vectors  $u$  and  $w$  such that  $x_2(t) = e^{4t}u + te^{4t}w$  is a second, linearly independent solution to the system.

(c) (10%) Write the general solution to the system.

4. (40%) Construct a matrix with the required property or say why that is impossible.

(a) (10%) Column space contains  $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$  and  $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$ , nullspace contains  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .

(b) (10%) Row space contains  $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$  and  $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$ , nullspace contains  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .

(c) (10%)  $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  has a solution and  $A^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .

(d) (10%) The columns add up to a column of 0s, the rows add to a row of 1s.