

國立臺北科技大學九十七學年度碩士班招生考試

系所組別：1521 1522 自動化科技研究所乙組

第一節 工程數學 試題

填准考證號碼

第一頁 共二頁

--	--	--	--	--	--	--	--	--	--

**注意事項：**

1. 本試題共十題，配分共 100 分。
2. 請標明大題、子題編號依序作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 作答須詳列計算或證明過程，並將答案清楚標示，否則不予計分。

1. (10%) Determine whether the following vectors are linearly dependent or independent.

$$(1) \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 7 \end{pmatrix}$$

$$(2) \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 11 \\ -6 \\ 12 \end{pmatrix}$$

2. (10%) Consider the system

$$2x_1 - 3x_2 + 5x_3 = 0$$

$$-x_1 + 7x_2 - x_3 = 0$$

$$4x_1 - 11x_2 + kx_3 = 0$$

For what value of  $k$  will the system have nontrivial solutions?

3. (10%) Find a linear transformation from two-dimensional vector space  $\mathbb{R}^2$  into the plane

$$W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : 2x - y + 3z = 0 \right\}$$

4. (10%) Find the rank of the matrix.

$$\begin{bmatrix} 1 & 2 & -1 & 3 & 1 \\ 0 & 1 & -3 & 2 & 3 \\ 2 & 3 & 1 & 4 & -1 \\ -1 & 2 & 2 & 2 & -5 \\ 3 & 1 & -1 & 2 & 4 \end{bmatrix}$$

5. (10%) Find the eigenvalues of the following matrix.

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -6 & 5 \end{bmatrix}$$

6. (10%) Given

$$A = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{bmatrix}$$

Find an orthonormal basis and a diagonal matrix  $D$  such that the linear transformation  $L$  defined by  $L(v) = Av$  is represented by  $D$  with respect to that basis.

注意：背面尚有試題

7. (10%) Let  $A$  be an  $m \times n$  matrix (where  $m < n$ ) whose rank is  $r$ .
- (1) What is the largest value  $r$  may be?
  - (2) How many vectors are in a basis for the row space of  $A$ ?
  - (3) How many vectors are in a basis for the column space of  $A$ ?
  - (4) Which vector space  $R^k$  has the row space as a subspace?
  - (5) Which vector space  $R^k$  has the column space as a subspace?

8. (10%) Given  $A = \begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$

Find the matrix  $P$  that diagonalizes  $A$ .

9. (10%) Prove or disprove the following statement.  
If a square matrix  $A$  is invertible, its inverse is unique.

10. (10%) Find the matrix norm  $\|A\|_\infty$ , where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$$