

國立臺北科技大學九十七學年度碩士班招生考試

系所組別：1511 1512 自動化科技研究所甲組

第一節 工程數學 試題

填准考證號碼

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第一頁 共一頁

注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、Solve the following problem by Laplace transform. (25%)

$$\begin{cases} \dot{x} = x + y \\ \dot{y} = 4x + y \end{cases}$$

$$x(0) = 1, y(0) = 0$$

二、Consider the differential operator $L = -d^2/dx^2$ with the boundary conditions $y(0) = y(L) = 0$, The eigenvalue problem is then

$$y'' + \lambda y = 0, \lambda > 0,$$

$$y(0) = y(L) = 0$$

1. solve the eigenvalue and eigenfunction and find the general solution of the differential equation (15%)
2. prove the eigenfunctions are mutually orthogonal. (10%) (Note: The inner product of two functions is defined as $(y_m, y_n) = \int_0^L y_m(x)y_n(x)dx$)

三、Consider the differential equation

$$xy'' + y' + xy = 0$$

If the solution can be assumed as: $y = x^\alpha \sum_{n=0}^{\infty} c_n x^n$, please find the serial solution of the above differential equation. (25%)

四、Describe the definition of an orthogonal matrix. Prove that the determinant of an orthogonal matrix must be 1 or -1. (10%)

五、Are the following column vectors linear independent in R^3 , if yes, explain why? If not, explain why not?

$$1. \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 12 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ -1 \end{bmatrix} \quad (5\%)$$

$$2. \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad (5\%)$$

$$3. \begin{bmatrix} 11 \\ -21 \\ 231 \end{bmatrix}, \begin{bmatrix} 13 \\ 45 \\ -21 \end{bmatrix}, \begin{bmatrix} 20 \\ 30 \\ 56 \end{bmatrix}, \begin{bmatrix} 77 \\ 90 \\ 15 \end{bmatrix} \quad (5\%)$$