國立臺北科技大學九十六學年度碩士班招生考試

系所組別：1511、1512、1521、1522 自動化科技研究所甲、乙組

第一節 工程數學 試題

注意事項：
1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Determine and explain why the following sets of vectors are linearly independent or linear dependent?

(a) $\{(2,3,1), (3,1,2), (1,2,3)\}$ (3%)

(b) $\{(1,0,1), (3,2,1), (1,4,2), (2,1,1)\}$ (3%)

(c) $\{(1,2,2,1), (1,2,2,0)\}$ (3%)

2. If $e^{i\omega} = \cos\omega + i\sin\omega$, A= $\begin{bmatrix} e^0 & e^{i\frac{2\pi}{3}} & e^{i\frac{4\pi}{3}} \\ e^{i\frac{4\pi}{3}} & e^0 & e^{i\frac{2\pi}{3}} \\ e^{i\frac{2\pi}{3}} & e^{i\frac{4\pi}{3}} & e^0 \end{bmatrix}$, determine the rank of A.

(6%)

3. If $\mathbf{x}=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{u}_1=\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{pmatrix}$, $\mathbf{u}_2=\begin{pmatrix} -\frac{\sqrt{6}}{4} \\ \frac{\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}$, $\mathbf{u}_3=\begin{pmatrix} -\frac{\sqrt{6}}{4} \\ \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$

(a) Please show that $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$ are orthonormal (10%)

(b) Express the vector **x** a as a linear combination of the basis $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$ (15%)

4. If $\mathbf{A}=\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathbf{B}=\begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

(a) please show whether **A**, and **B** are diagonal matrices or not? (5%)

(b) compute $(\mathbf{AB})^{-1}$=? (5%)

5. If a matrix $\mathbf{A}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

(a) Find all the eigenvalues of the matrix **A** (7%)

(b) Find the corresponding linearly independent eigenvectors of the matrix **A** (8%)

(c) Find the exp(At), i.e. the exponential of the matrix $\mathbf{A}t = \begin{bmatrix} t & 0 & 0 \\ 0 & 0 & 2t \\ 0 & 2t & 0 \end{bmatrix}$ (10%)

6. If a vector space $\mathbf{V}=\{(a_1, a_2, \ldots\ldots, a_n) \mid a_i \in \boldsymbol{R}, \sum_{i=1}^{n} i a_i = 0\}$,

(a) Find a basis of the vector space. Show they are a linearly independent set of vectors in **V** which spans the space **V**.) (20%)

(b) What is the dimension of the vector space **V**. (5%)