國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1521、1522 自動化科技研究所乙組

第一節 工程數學 試題

填 准 考 證 號 碼

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1.(10%) Which of the following are spanning sets for R^3?

(1) $\{(1,1,1)^T, (1,1,0)^T, (1,0,0)^T\}$.

(2) $\{(1,2,4)^T, (2,1,3)^T, (4,-1,1)^T\}$.

2. (20%) Find the eigenvalues and the corresponding eigenspaces.

(1)

$$A=\begin{bmatrix}2 & -3 & 1\\ 1 & -2 & 1\\ 1 & -3 & 2\end{bmatrix}.$$

(2)

$$A=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$$

3.(20%) Shows the following operations are isotropic (invariant to rotation).

(1) The Laplacian operation

$$\nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$$

(2) The magnitude of the gradient

$$|\nabla f|=mag(\nabla f)=\left[G_x^2+G_y^2\right]^{1/2}=\left[\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

4.(20%) Let

$$A=\begin{bmatrix}1 & 1\\ 1 & 1\\ 0 & 0\end{bmatrix}$$

Compute the singular values and the singular value decomposition of A.

5.(15%) Find least square solution of the given linear system

$$2x_1 + x_2 = 26$$
$$x_1 = -13$$
$$2x_1 + 3x_2 = 0$$

6.(15%) Find the fundamental matrix e^{At} for the system

$$X' = AX \text{, where } A=\begin{bmatrix}1 & 0 & 0\\ 1 & 3 & 0\\ 0 & 1 & 1\end{bmatrix}.$$