國立臺北科技大學

九十四學年度自動化科技研究所入學考試

工程數學乙組試題

填准考證號碼　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. (20%) Linearly dependent or linearly independent

   (1) Show that $(4, 2, 3)^T$, $(2, 3, 1)^T$, and $(2, -5, 3)^T$ are linearly dependent or linearly independent.

   (2) Show that e^x and e^{-x} are linearly dependent or linearly independent in $C(-\infty, \infty)$.

   (3) Show that x^2 and $x|x|$ are linearly dependent or linearly independent in $C[-1, 1]$.

   (4) Show that $1, x, x^2, x^3$ are linearly dependent or linearly independent.

2. (20%) Linear transformation

   (1) Suppose that in P_3 we want to change from the ordered basis $[1, x, x^2]$ to the ordered basis $[1, 2x, 4x^2 - 2]$.

   (2) Given any $p(x) = a + bx + cx^2$ in P_3, to find the coordinates of $p(x)$ with respect to $[1, 2x, 4x^2 - 2]$.

3. (20%) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are any two vectors in an inner product space V, then

$$|\langle u, v\rangle| \leq \|u\| \|v\|$$

Equality holds if and only if $\boldsymbol{u}$ and $\boldsymbol{v}$ are linearly dependent. Please prove this Cauchy-Schwarz theorem.

4. (20%) In a certain town 30 percent of the married women get divorced each year and 20 percent of the single women get married each year. There are 8000 married women and 2000 single women and the total population remains constant. Find the number of married women and single women after 5 years. What will be the long-range prospects if these percentages of marriages and divorces continue indefinitely into the future?

5. (20%) Prove or disprove the following statements:
   (1) Let S be the subspace of R^2 spanned by e_1 and let T be the subspace of R^2 spanned by e_2. Then $S \bigcup T$ is subspace of R^2.
   (2) If A is one R^2 matrices. $det\ A^{-1} = (det\ A)^{-1}$.
   (3) If A and B are two R^2 matrices. $(A+B)^{-1} = A^{-1} + B^{-1}$.
   (4) Similar matrices have the same eigenvalues and eigenvectors.
   (5) If $A^T = A^{-1}$, then $det\,(A) = 1$.
   (6) If A is an $n \times n$ orthogonal matrix, then rank $A < n$.
   (7) Two vectors in R^3 always span a two-dimensional subspace.
   (8) Let V be an n-dimensional vector space. If a set of m vectors spans V, then m=n.
   (9) Consider $Ax = b$ where A is $m \times n$. If the rank of matrix A is n, then there is a solution.
   (10) Consider $Ax = b$ where A is $m \times n$. If $rank(A) = min(n, m)$, then there exists a least squares solution by $x = (A^T A)^{-1} A^T b$.