國立臺北科技大學

九十三學年度自動化科技研究所入學考試

工程數學(乙組)試題

填准考證號碼　　　　　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共五題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. (25%)For vector space
   (1) Let $v_1, v_2, \cdots, v_m$ are the vectors of vector space V. Explain that linearly dependent and linearly independent for $v_1, v_2, \cdots, v_m$.
   (2) Are $cos(x)$ and $sin(x)$ linearly dependent or linearly independent? Why?
   (3) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$, $D = \begin{bmatrix} -1 & -2 \\ 0 & -2 \end{bmatrix}$. Are A, B, C, D, linearly dependent or linearly independent? Why?

2. (15%)Let A be $n \times n$ matrix, $A = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ 0 & 1 & \cdots & 0 & -a_2 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -a_{n-2} \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix}$ prove that the characteristic polynomial of A is $det(A - \lambda I) = (-1)^n \left(\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_2\lambda^2 + a_1\lambda + a_0\right)$.

3. (20%) $A = \begin{bmatrix} 4 & 1+i \\ 1-i & 4 \end{bmatrix}$, $B = e^{A}$

(1) Find the eigenvalues and eigenvectors of A, and det(A).

(2) Find the eigenvalues and eigenvectors of B, and det(B).

4. (10%)Let $T : R^2 \to R^3$ be a linear transformation such that
$T(x_1, x_2) = (2x_1 + 3x_2, -2x_1 - 4x_2, -6x_1 - 5x_2)$
find $x = (x_1, x_2)$, such that $T(x) = (-4, 7, 0)$

5. (30%)Let $S = Span\left[(1\ \ 3\ \ 1\ \ 1)^T, (1\ \ 1\ \ 1\ \ 1)^T, (-1\ \ 5\ \ 2\ \ 2)^T\right]$ be a subspace of R^4,

and let $b = (4, -1, 5, 1)^T$

(1) Find an orthonormal basis for S.

(2) Use your answer in (1) to find the projection p of b onto S.

(3) Given $A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & 5 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$. Use your answer in (2) to solve the least squares problem

$Ax = b$.