國立臺北科技大學 102 學年度碩士班招生考試

系所組別：2140、2150 電機工程系碩士班丁、戊組

第二節　工程數學　試題

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1.Assume the cumulative distribution function (CDF) of a random variable T is given by

$$F_T(t)=\begin{cases}1-e^{-t/3}, & t\geq 0\\ 0 & ,\text{otherwise}\end{cases}$$

(10%) (a)Find the probability density function (PDF) of the random variable T

(5%) (b)Find $P[2\leq T<4]$

(5%) (c)Find $E[T]$

2. Let U be a uniform (0,1) random variable.

(10%) (a) Assume $g(x)$ be an arbitrary function with an inverse $g^{-1}(u)$ defined for $0<u<1$. Let random variable $X = g^{-1}(U)$. Show that the cumulative distribution function (CDF) of X is $F_X(x) = g(x)$.

(10%) (b)Derive $f(U)$ such that $X=f(U)$ is the exponential (1) random variable.

3. (10%) A modem transmits one million bits. Each bit is 0 or 1 independently with equal probability. Let X_i be the value of bit i (either 0 or 1). The number of ones in one million bits is $W=\sum_{i=0}^{10^6} X_i$. Use the central limit theorem to approximate the probability of at least 502,000 ones in the transmission, i.e., $P[W\geq 502000]=?$
[Express your answer using the standard normal CDF $\Phi(.)$]

4. Given the vectors $\mathbf{y}=\begin{bmatrix}4\\3\\3\\-1\end{bmatrix}, \mathbf{u}_1=\begin{bmatrix}1\\1\\0\\1\end{bmatrix}, \mathbf{u}_2=\begin{bmatrix}-1\\3\\1\\-2\end{bmatrix}, \mathbf{u}_3=\begin{bmatrix}-1\\0\\1\\1\end{bmatrix}$. Let W be the subspace spanned by $\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3\}$.

(10%) (a) Represent $\mathbf{y}$ as the sum of a vector $\mathbf{y}_1$ in W and a vector $\mathbf{y}_2$ in the orthogonal complement of W.

(5%) (b) Determine whether the matrix $A=[\mathbf{y}\ \ \mathbf{u}_1\ \ \mathbf{u}_2\ \ \mathbf{u}_3]$ is invertible, and justify your answer.

5. Let $B=\begin{bmatrix}1&0&2&0\\1&1&1&0\\1&1&1&1\\1&1&1&1\end{bmatrix}$.

(5%) (a) Find a basis for the column space of B.

(15%) (b) Find an orthogonal basis for the column space of B.

(5%) (c) Determine whether the number 0 is an eigenvalue of B, and justify your answer.

6. (10%) Determine the dimensions of the eigenspaces corresponding to the eigenvalues of the matrix $C=\begin{bmatrix}1&-2&0&0\\-2&1&0&0\\0&0&1&-2\\0&0&-2&1\end{bmatrix}$.