

國立臺北科技大學 100 學年度碩士班招生考試

系所組別：3510 化學工程研究所甲組

第三節 工程數學 試題

第一頁 共一頁

注意事項：

1. 本試題共 6 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Consider the differential equation $x \frac{dy}{dx} - 2x^2y + 2y \ln y = 0$.

- (a) Solve $y(x)$ by the substitution of $u = \ln y$. (10%)
- (b) Find an integration factor $I(x,y)$ so that the multiplication of the differential equation by $I(x,y)$ yields an exact differential equation. (5%)

2. Find the general solution of $(x-1)^2 y'' + (x-1)y' + y = x$. (15%)

The following integral formulas may be useful:

$$\int \cos(\ln x) dx = \frac{x}{2} [\sin(\ln x) + \cos(\ln x)]; \quad \int \sin(\ln x) dx = \frac{x}{2} [\sin(\ln x) - \cos(\ln x)]$$

3. Let $\mathbf{A} = \begin{bmatrix} 3 & \sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ and $\mathbf{B} = \mathbf{A}^5 - 3\mathbf{A}^4 - \mathbf{A} + 4\mathbf{I}$, where \mathbf{I} is the 2×2 identity matrix.

- (a) Find the eigenvalues and corresponding eigenvectors of \mathbf{B} . (12%)
- (b) Calculate $\mathbf{B}^{0.5}$. (8%)

4. Let $F(s)$ be the Laplace transform of function $f(t)$, and $F(s) = \frac{1}{s} \left[1 - \frac{-s+1}{(s+1)^2} e^{-2s} \right]$.

Find the value of $\int_0^{\infty} |f(t)| dt$. (15%)

5. Let $\vec{F} = y\vec{i} + (x-2xz)\vec{j} - xy\vec{k}$, and S is the surface of the hemispherical region bounded by a sphere $x^2 + y^2 + z^2 = R^2$ for $z > 0$.

- (a) Calculate $\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dS$, where \vec{n} is the outward normal vector. (10%)
- (b) Calculate the line integral $\int_C [y dx + (x-2xz) dy - xy dz]$ along C clockwise, where C is the boundary of S on the $x-y$ plane. (5%)

6. Solve the following partial differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{for } 0 \leq x \leq 2, \quad 0 \leq y \leq 2$$

with boundary conditions $u(0,y) = u(2,y) = u(x,0) = 0$ and $u(x,2) = 1$. (20%)