國立臺北科技大學 100 學年度碩士班招生考試

系所組別：4211、4212 經營管理系碩士班甲組

第一節　統計學　試題

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Suppose that the two dimensional random variable (X,Y) is uniformly distributed over R, where R is defined by $\{(x,y) \mid x^2+y^2 \le 1, y \ge 0\}$. Please evaluate ρ_{xy}, the correlation coefficient. (15%)

2. Use the moment generating function to show that if X and Y are independent random variables with distribution $N(\mu_x,\sigma_x^2)$ and $N(\mu_y,\sigma_y^2)$, respectively, then $Z=aX+bY$ is again normally distributed, where a and b are constants. (15%)

3. Suppose that X has distribution N(0, 0.09). A sample of size 25 is obtained from X, say $X_1, \ldots X_{25}$. What is the probability that $\sum_{i=1}^{25} X_i^2$ exceeds 1.5? (15%)

4. A random variable X has pdf $f(x)=(\beta+1)x^{\beta},\ 0<x<1$.
   (a) Please obtain the ML estimate of β, based on a sample $X_1, \ldots, X_n$. (10%)
   (b) Please evaluate the estimate if the sample values are 0.3, 0.8, 0.27, 0.35, 0.6, and 0.55. (10%)

5. Suppose that X has a distribution $N(\mu,\sigma^2)$, where μ and σ are unknown. A sample of size 15 yielded the values $\sum_{i=1}^{15} X_i = 8.7$ and $\sum_{i=1}^{15} X_i^2 = 27.3$. Please find a (two -sided) 95 percent confidence interval for σ^2. (15%)

6. If X, Y and Z are three random variables, two kinds of relationship might exist among them, (i) X, Y and Z are mutually dependent; (ii) X, Y and Z have multi-collinearity. Please explain how to identify the these relationships, what is the difference between these relationships, what kind of problem might caused by these relationships, and how to solve these problems. (20%)

$$P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

Standard normal distribution table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0.0	0.50	0.50	0.49	0.49	0.48	0.48	0.48	0.47	0.47	0.46
-0.1	0.46	0.46	0.45	0.45	0.44	0.44	0.44	0.43	0.43	0.42
-0.2	0.42	0.42	0.41	0.41	0.41	0.40	0.40	0.39	0.39	0.39
-0.3	0.38	0.38	0.37	0.37	0.37	0.36	0.36	0.36	0.35	0.35
-0.4	0.34	0.34	0.34	0.33	0.33	0.33	0.32	0.32	0.32	0.31
-0.5	0.31	0.31	0.30	0.30	0.29	0.29	0.29	0.28	0.28	0.28
-0.6	0.27	0.27	0.27	0.26	0.26	0.26	0.25	0.25	0.25	0.25
-0.7	0.24	0.24	0.24	0.23	0.23	0.23	0.22	0.22	0.22	0.21
-0.8	0.21	0.21	0.21	0.20	0.20	0.20	0.19	0.19	0.19	0.19
-0.9	0.18	0.18	0.18	0.18	0.17	0.17	0.17	0.17	0.16	0.16
-1	0.16	0.16	0.15	0.15	0.15	0.15	0.14	0.14	0.14	0.14
-1.1	0.14	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.12	0.12
-1.2	0.12	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.10
-1.3	0.10	0.10	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08
-1.4	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07
-1.5	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
-1.6	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
-1.7	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04

Chi-square distribution table

df	0.995	0.99	0.975	0.95	0.9	0.8	0.2	0.1	0.05	0.025	0.01	0.005
11	2.60	3.05	3.82	4.57	5.58	6.99	14.63	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	7.81	15.81	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	8.63	16.98	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	9.47	18.15	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	10.31	19.31	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	11.15	20.47	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	12.00	21.61	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	12.86	22.76	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	13.72	23.90	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	14.58	25.04	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	15.44	26.17	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	16.31	27.30	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	17.19	28.43	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	18.06	29.55	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	18.94	30.68	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	19.82	31.79	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	20.70	32.91	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	21.59	34.03	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	22.48	35.14	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	23.36	36.25	40.26	43.77	46.98	50.89	53.67