注意事項：

1. 本試題共七題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、

Given a system $Ax = b$, where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 3 \\ 2 & -2 & a \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Find the value of a that makes the given system inconsistent. (10%)

二、

Determine the least squares solution to $Ax = b$, where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}.$$

(10%)

三、

The matrix $A = \begin{bmatrix} 2 & 3 & 0 & 1 \\ 4 & 5 & 3 & 3 \\ -2 & -6 & 7 & 7 \\ 8 & 9 & 5 & 21 \end{bmatrix}$ has a LU-factorization, i.e., $A = LU$. Please find the matrix L and U, where L is a lower triangular matrix with its diagonal entries equal to 1, and U is an upper triangular matrix. (15%)

四、

Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{bmatrix}$.

(a). Find the eigenvalues of A and the corresponding eigenvectors. (10%)

(b). Is matrix A diagonalizable? That is, can we find a nonsingular matrix S and a diagonal matrix D such that $S^{-1}AS = D$? If the answer is "Yes", find the resulted diagonal matrix D and the nonsingular matrix S that diagonalizes A. On the other hand, give the reason if your answer is "No". (10%)

五、

The joint probability density function (p.d.f.) of the two random variables X and Y is given by

$$f(x,y) = \begin{cases} cxy & 0 \le x, y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a). Find the constant c. (5%)

(b). Find the marginal p.d.f. of X and Y, i.e., to find $f_x(x)$ and $f_y(y)$. (8%)

(c). Are the two random variables X and Y independent? Prove your answer. (4%)

(d). Find the mean and variance of X. (8%)

六、

Consider the game of throwing a fair dice five times. Find the following probabilities.

(a). The probability that the sixth-point occurs three times during the five trials. (5%)

(b). The sixth-point occurs one time in the first trial given that the sixth-point occurs three times during the five trials. (5%)

七、

Let X be a random variable that denotes the score of students in the subject "Engineering Mathematics". It is known that X has a mean $E[X] = \mu = 40$ and variance $\sigma^2 = 4$. Determine the two parameters a and b, such that the mean and variance after the linear transformation $(aX+b)$ are **60** and **16**, respectively. (10%)