

115PM01

# 國立臺北科技大學 115 學年度碩士班招生考試

系所組別：4100 工業工程與管理系碩士班

## 第一節 統計學 試題

第 1 頁 共 2 頁

### 注意事項：

1. 本試題共分選擇題(10 題，共 30 分)、是非題(10 題，共 20 分)及計算題(4 題，共 50 分)等三部分，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

Part I: Multiple-Choice (10 questions, each for 3 points. Total 30 points)

1. If A and B are independent with  $P(A)=0.2$  and  $P(B)=0.5$ , what is  $P(A|B)$ ?  
A. 0.1    B. 0.2    C. 0.4    D. 0.7
2. If  $E[X]=\mu$  and  $\text{Var}(X)=\sigma^2$ , what is  $\text{Var}(aX+b)$ ?  
A.  $a\sigma^2$     B.  $a^2\sigma^2$     C.  $\sigma^2+b^2$     D.  $(a+b)^2\sigma^2$
3. If  $Z \sim N(0,1)$ , then  $Z^2$  follows: A.  $\chi^2(1)$     B.  $t(1)$     C.  $\chi^2(2)$     D.  $\text{Exp}(1)$
4. The correct interpretation of a 95% confidence interval is:  
A.  $\theta$  has 95% probability to be inside    B. 95% of such intervals cover  $\theta$  in repeated sampling  
C. sample is drawn with prob 0.95    D. p-value equals 0.05
5. Let  $X \sim \text{Geometric}(p)$  be the number of trials until the first success (support  $\{1,2,\dots\}$ ). What is  $E[X]$ ? A.  $1/p$     B.  $(1-p)/p$     C.  $p$     D.  $1/(1-p)$
6. An urn has 8 red and 12 blue balls. Draw 5 without replacement. What is  $P(\text{exactly 2 red})$ ? A.  $C(8,2)C(12,3)/C(20,5)$     B.  $C(5,2)(8/20)^2(12/20)^3$     C.  $(8/20)^2(12/20)^3$   
D.  $C(20,2)/C(20,5)$
7. In a  $\chi^2$  goodness-of-fit test with k categories and m parameters estimated from data (e.g., Poisson with  $\lambda$  estimated), the df is: A.  $k-1$     B.  $k-m$     C.  $k-1-m$     D.  $k-2m$
8. In a pooled two-sample t confidence interval for  $\mu_1-\mu_2$  (equal variances, normal), the degrees of freedom is: A.  $n_1+n_2$     B.  $n_1+n_2-1$     C.  $n_1+n_2-2$     D.  $\min(n_1, n_2)-1$
9. In simple linear regression  $y=\beta_0+\beta_1x+\varepsilon$ , the test for  $H_0: \beta_1=0$  uses a statistic that follows: A.  $\chi^2(n-1)$     B.  $t(n-2)$     C.  $F(1, n-1)$     D.  $N(0,1)$  regardless of n
10. In multiple regression with p predictors, the overall F test for  $H_0: \beta_1=\dots=\beta_p=0$  has degrees of freedom: A.  $(p, n-p-1)$     B.  $(n-1, p)$     C.  $(p-1, n-p)$     D.  $(1, n-2)$

Part II: True/False (10 questions, each for 2 points. Total 20 points)

1. If two events are independent, then they must be mutually exclusive.
2. ANOVA can be viewed as a special case of a linear model using dummy variables.
3. A t distribution has heavier tails than the standard normal.
4. The CLT says any distribution becomes normal for large samples.
5. With right-skewed data and outliers, the median is typically more robust than the mean.
6. An F distribution can be expressed as the ratio of two independent chi-square variables divided by their degrees of freedom.
7. If  $E[X]$  exists, then  $\text{Var}(X)$  must exist.
8. At fixed  $\alpha$ , if the rejection region becomes more stringent (harder to reject),  $\beta$  typically increases.
9. If the overall F test is not significant ( $p>\alpha$ ), then every individual coefficient must be non-significant.
10. For the same  $x_0$ , a prediction interval is always narrower than a CI for the mean response.

Part III: Calculation & Application (4 problems, Total 50 points)

1. To test for any significant difference in the number of hours between breakdowns for four machines, the following data were obtained. (12 points)

Machine 1	Machine 2	Machine 3	Machine 4
6.4	8.7	11.1	9.9
7.8	7.4	10.3	12.8
5.3	9.4	9.7	12.1
7.4	10.1	10.3	10.8
8.4	9.2	9.2	11.3
7.3	9.8	8.8	11.5

- a. At the  $\alpha = 0.05$  level of significance, what is the difference, if any, in the population mean times among the four machines? (3 points)
- b. Use Fisher's LSD procedure to test for the equality of the means for machines 2 and 4. Use a 0.05 level of significance. (3 points)
- c. Use the Bonferroni adjustment to test for a significant difference between all pairs of means. Assume that a maximum overall experimentwise error rate of 0.05 is desired. (6 points)

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2. A production line operation is tested for filling weight accuracy (in grams) using the following hypotheses. (13 points)

Hypothesis	Conclusion and Action
$H_0: \mu = 16$	Filling okay; keep running
$H_a: \mu \neq 16$	Filling off standard; stop and adjust machine

The sample size is 30 and the population standard deviation is  $\sigma = 0.8$ . Use  $\alpha = 0.05$

- What would a Type II error mean in this situation? (3 points)
  - What is the probability of making a Type II error when the machine is overfilling by 0.5 grams? (3 points)
  - What is the power of the statistical test when the machine is overfilling by 0.5 grams? (3 points)
  - Show the power curve for this hypothesis test. What information does it contain for the production manager? (4 points)
3. A salesperson makes four calls per day. A sample of 100 days gives the following frequencies of sales volumes. (12 points)

Number of Sales	Observed Frequency (days)
0	30
1	32
2	25
3	10
4	3
Total	100

Records show sales result from 30% of all sales calls. Assuming independent sales calls, the number of sales per day should follow a binomial probability distribution. The binomial probability function is

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

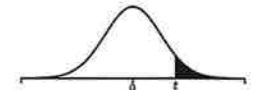
For this exercise, assume that the population has a binomial probability distribution with  $n = 4$ ,  $p = .30$ , and  $x = 0, 1, 2, 3$ , and 4.

- Compute the expected frequencies for  $x = 0, 1, 2, 3$ , and 4 by using the binomial probability function. Combine categories if necessary to satisfy the requirement that the expected frequency is five or more for all categories. (6 points)

- Use the goodness of fit test to determine whether the assumption of a binomial probability distribution should be rejected. Use  $\alpha = 0.05$ . Because no parameters of the binomial probability distribution were estimated from the sample data, set the degrees of freedom to  $k - 1$  when  $k$  is the number of categories. (6 points)
4. A statistical professor at Givens College is interested in the relationship between hours spent studying and total points earned in a course. Data collected on 10 students who took the course last quarter follow. (13 points)

Hours Spent Studying	Total Points Earned
45	40
30	35
90	75
60	65
105	90
65	50
90	90
80	80
55	45
75	65

- Develop an estimated regression equation showing how total points earned is related to hours spent studying. (5 points)
- Test the significance of the model with  $\alpha = 0.05$ . (3 points)
- Predict the total points earned by Eric Chang. He spent 95 hours studying. (2 points)
- Develop a 95% prediction interval for the total points earned by Eric Chang. (3 points)



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

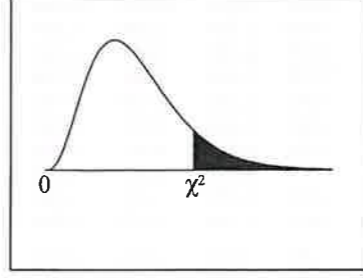
Critical Values for Student's *t*-Distribution.

df	Upper Tail Probability: Pr( <i>T</i> > <i>t</i> )									
	0.2	0.1	0.05	0.04	0.03	0.025	0.02	0.01	0.005	0.0005
1	1.376	3.078	6.314	7.916	10.579	12.706	15.895	31.821	63.657	636.619
2	1.061	1.886	2.920	3.320	3.896	4.303	4.849	6.965	9.925	31.599
3	0.978	1.638	2.353	2.605	2.951	3.182	3.482	4.541	5.841	12.924
4	0.941	1.533	2.132	2.333	2.601	2.776	2.999	3.747	4.604	8.610
5	0.920	1.476	2.015	2.191	2.422	2.571	2.757	3.365	4.032	6.869
6	0.906	1.440	1.943	2.104	2.313	2.447	2.612	3.143	3.707	5.959
7	0.896	1.415	1.895	2.046	2.241	2.365	2.517	2.998	3.499	5.408
8	0.889	1.397	1.860	2.004	2.189	2.306	2.449	2.896	3.355	5.041
9	0.883	1.383	1.833	1.973	2.150	2.262	2.398	2.821	3.250	4.781
10	0.879	1.372	1.812	1.948	2.120	2.228	2.359	2.764	3.169	4.587
11	0.876	1.363	1.796	1.928	2.096	2.201	2.328	2.718	3.106	4.437
12	0.873	1.356	1.782	1.912	2.076	2.179	2.303	2.681	3.055	4.318
13	0.870	1.350	1.771	1.899	2.060	2.160	2.282	2.650	3.012	4.221
14	0.868	1.345	1.761	1.887	2.046	2.145	2.264	2.624	2.977	4.140
15	0.866	1.341	1.753	1.878	2.034	2.131	2.249	2.602	2.947	4.073
16	0.865	1.337	1.746	1.869	2.024	2.120	2.235	2.583	2.921	4.015
17	0.863	1.333	1.740	1.862	2.015	2.110	2.224	2.567	2.898	3.965
18	0.862	1.330	1.734	1.855	2.007	2.101	2.214	2.552	2.878	3.922
19	0.861	1.328	1.729	1.850	2.000	2.093	2.205	2.539	2.861	3.883
20	0.860	1.325	1.725	1.844	1.994	2.086	2.197	2.528	2.845	3.850
21	0.859	1.323	1.721	1.840	1.988	2.080	2.189	2.518	2.831	3.819
22	0.858	1.321	1.717	1.835	1.983	2.074	2.183	2.508	2.819	3.792
23	0.858	1.319	1.714	1.832	1.978	2.069	2.177	2.500	2.807	3.768
24	0.857	1.318	1.711	1.828	1.974	2.064	2.172	2.492	2.797	3.745
25	0.856	1.316	1.708	1.825	1.970	2.060	2.167	2.485	2.787	3.725
26	0.856	1.315	1.706	1.822	1.967	2.056	2.162	2.479	2.779	3.707
27	0.855	1.314	1.703	1.819	1.963	2.052	2.158	2.473	2.771	3.690
28	0.855	1.313	1.701	1.817	1.960	2.048	2.154	2.467	2.763	3.674
29	0.854	1.311	1.699	1.814	1.957	2.045	2.150	2.462	2.756	3.659
30	0.854	1.310	1.697	1.812	1.955	2.042	2.147	2.457	2.750	3.646
31	0.853	1.309	1.696	1.810	1.952	2.040	2.144	2.453	2.744	3.633
32	0.853	1.309	1.694	1.808	1.950	2.037	2.141	2.449	2.738	3.622
33	0.853	1.308	1.692	1.806	1.948	2.035	2.138	2.445	2.733	3.611
34	0.852	1.307	1.691	1.805	1.946	2.032	2.136	2.441	2.728	3.601
35	0.852	1.306	1.690	1.803	1.944	2.030	2.133	2.438	2.724	3.591
36	0.852	1.306	1.688	1.802	1.942	2.028	2.131	2.434	2.719	3.582
37	0.851	1.305	1.687	1.800	1.940	2.026	2.129	2.431	2.715	3.574
38	0.851	1.304	1.686	1.799	1.939	2.024	2.127	2.429	2.712	3.566
39	0.851	1.304	1.685	1.798	1.937	2.023	2.125	2.426	2.708	3.558
40	0.851	1.303	1.684	1.796	1.936	2.021	2.123	2.423	2.704	3.551
41	0.850	1.303	1.683	1.795	1.934	2.020	2.121	2.421	2.701	3.544
42	0.850	1.302	1.682	1.794	1.933	2.018	2.120	2.418	2.698	3.538
43	0.850	1.302	1.681	1.793	1.932	2.017	2.118	2.416	2.695	3.532
44	0.850	1.301	1.680	1.792	1.931	2.015	2.116	2.414	2.692	3.526
45	0.850	1.301	1.679	1.791	1.929	2.014	2.115	2.412	2.690	3.520
46	0.850	1.300	1.679	1.790	1.928	2.013	2.114	2.410	2.687	3.515
47	0.849	1.300	1.678	1.789	1.927	2.012	2.112	2.408	2.685	3.510
48	0.849	1.299	1.677	1.789	1.926	2.011	2.111	2.407	2.682	3.505
49	0.849	1.299	1.677	1.788	1.925	2.010	2.110	2.405	2.680	3.500
50	0.849	1.299	1.676	1.787	1.924	2.009	2.109	2.403	2.678	3.496
60	0.848	1.296	1.671	1.781	1.917	2.000	2.099	2.390	2.660	3.460
70	0.847	1.294	1.667	1.776	1.912	1.994	2.093	2.381	2.648	3.435
80	0.846	1.292	1.664	1.773	1.908	1.990	2.088	2.374	2.639	3.416
90	0.846	1.291	1.662	1.771	1.905	1.987	2.084	2.368	2.632	3.402
100	0.845	1.290	1.660	1.769	1.902	1.984	2.081	2.364	2.626	3.390
120	0.845	1.289	1.658	1.766	1.899	1.980	2.076	2.358	2.617	3.373
140	0.844	1.288	1.656	1.763	1.896	1.977	2.073	2.353	2.611	3.361
180	0.844	1.286	1.653	1.761	1.893	1.973	2.069	2.347	2.603	3.345
200	0.843	1.286	1.653	1.760	1.892	1.972	2.067	2.345	2.601	3.340
500	0.842	1.283	1.648	1.754	1.885	1.965	2.059	2.334	2.586	3.310
1000	0.842	1.282	1.646	1.752	1.883	1.962	2.056	2.330	2.581	3.300
∞	0.842	1.282	1.645	1.751	1.881	1.960	2.054	2.326	2.576	3.291
	60%	80%	90%	92%	94%	95%	96%	98%	99%	99.9%
	Confidence Level									

Note:  $t(\infty)_{\alpha/2} = Z_{\alpha/2}$  in our notation.

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Chi-Square Distribution Table



The shaded area is equal to  $\alpha$  for  $\chi^2 = \chi^2_{\alpha}$ .

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

F Distribution Table

d.f.D.	$\alpha = 0.05$																		
	d.f.N.																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00