115CC03

國立臺北科技大學 115 學年度碩士班招生考試

系所組別：2230 電子工程系碩士班丙組

第一節 電磁學 試題

注意事項：

1.本試題共四題，第一題 20 分，第二題 30 分，第三題 30 分，第四題 20 分，共 100 分。
2.不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3.全部答案均須在答案卷之答案欄內作答，否則不予計分。

List of symbols:

- $\vec{E}$: electric field intensity (V/m)
- $\vec{H}$: magnetic field intensity (A/m)
- $\vec{D}$: electric flux density (C/m^2)
- $\vec{B}$: magnetic flux density (T)
- $\vec{J}$: volumetric electric current density (A/m^2)
- ρ: electric charge density (C/m^3)

Vector identities:

- $\vec{\nabla}(fg) = g\vec{\nabla}f + f\vec{\nabla}g$
- $\vec{\nabla}\cdot(f\vec{a}) = \vec{\nabla}f\cdot\vec{a} + f\vec{\nabla}\cdot\vec{a}$
- $\vec{\nabla}\times(f\vec{a}) = \vec{\nabla}f\times\vec{a} + f\vec{\nabla}\times\vec{a}$
- $\nabla^2\vec{a} = \vec{\nabla}(\vec{\nabla}\cdot\vec{a}) - \vec{\nabla}\times(\vec{\nabla}\times\vec{a})$

1. Vector algebra.

(a) Show that

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c}),$$

where $\vec{a},\vec{b},\vec{c},\vec{d}$ are arbitrary vectors. (10%)

(b) Show that

$$\hat{r}\times(\hat{r}\times\vec{a}) = -\hat{\theta}(\hat{\theta}\cdot\vec{a}) - \hat{\phi}(\hat{\phi}\cdot\vec{a}),$$

where $(\hat{r},\hat{\theta},\hat{\phi})$ are the unit vectors in spherical coordinate system, and $\vec{a}$ is an arbitrary vector. (10%)

2. Maxwell equations.

(a) Write the Maxwell equations and the associated continuity relations in differential form for both the time domain and the frequency domain (phasor, in $e^{i\omega t}$ convention, $\omega = 2\pi f$ being the angular frequency). (10%)

(b) Starting from Maxwell equations in the frequency domain, show that in a linear, isotropic, and homogeneous medium with permittivity ϵ and permeability μ with the constitutive relations $\vec{D} = \epsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$, the electric field satisfies

$$\nabla^2\vec{E} + k^2\vec{E} = \vec{\nabla}\left(\frac{\rho}{\epsilon}\right) + i\omega\mu\vec{J},$$

where $k^2 = \omega^2\mu\epsilon$ is the wavenumber. (10%)

(c) Consider a source-free region with nonhomogeneous permittivity ϵ (i.e., $\epsilon = \epsilon(\vec{r})$) and homogeneous permeability μ. Show that the electric field satisfies

$$\nabla^2\vec{E} + k^2\vec{E} = -\vec{\nabla}\left(\vec{E}\cdot\vec{\nabla}\ln\epsilon\right).$$

(10%)

3. Plane-wave solution and related topics.

(a) From the plane–wave solution in an unbounded, source-free, lossless homogeneous medium, show that for divergence and curl, the differential operator $\vec{\nabla}$ can be replaced by $-i\vec{k}$ when acting on fields of the form

$$\vec{E} = \vec{E}_0 e^{-i\vec{k}\cdot\vec{r}}, \qquad \vec{H} = \vec{H}_0 e^{-i\vec{k}\cdot\vec{r}},$$

where $\vec{k} = k\hat{k}$ is the wavevector and $\vec{E}_0$, $\vec{H}_0$ are constant vectors. (10%)

(b) Show that $\vec{E}_0$, $\vec{H}_0$, and $\vec{k}$ are perpendicular to each other. (5%)

(c) In a lossy medium with $\epsilon = \epsilon' - i\epsilon''$ ($\epsilon',\epsilon'' \in \mathbb{R}^+$), show that the wavenumber can be written in the form $k = \beta - i\alpha$ (suppose that $\alpha,\beta \in \mathbb{R}^+$), where

$$\beta = \omega\left\{\frac{\mu\epsilon'}{2}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1\right]\right\}^{\frac{1}{2}},$$

$$\alpha = \omega\left\{\frac{\mu\epsilon'}{2}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1\right]\right\}^{\frac{1}{2}}.$$

(10%)

(d) Show that

$$v_g = \frac{v_p}{1 - \dfrac{\omega}{v_p}\dfrac{dv_p}{d\omega}},$$

where v_g is the group velocity and v_p is the phase velocity. (5%)

注意：背面尚有試題

4. Geometrical-optics approximation of Maxwell equations.

(a) In a source-free linear, isotropic, homogeneous medium with permittivity ϵ and permeability μ, suppose that the solution for Maxwell equations can be written in the form of

$$\vec{E} = \vec{E}_0(\vec{r})e^{-ik_0\psi(\vec{r})}, \qquad \vec{H} = \vec{H}_0(\vec{r})e^{-ik_0\psi(\vec{r})},$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the free-space wavenumber. Show that, in the limit of $k_0 \to \infty$, we get

$$\vec{\nabla}\psi \times \vec{E}_0 = c\mu\vec{H}_0,$$
$$\vec{\nabla}\psi \times \vec{H}_0 = -c\epsilon\vec{E}_0,$$
$$\vec{\nabla}\psi \cdot \vec{E}_0 = 0,$$
$$\vec{\nabla}\psi \cdot \vec{H}_0 = 0,$$

where $c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ is the speed of light in vacuum. (10%)

(b) Show that

$$\vec{\nabla}\psi \cdot \vec{\nabla}\psi = n^2,$$

where $n = \sqrt{\mu\epsilon/(\mu_0\epsilon_0)}$ is the refractive index of the medium. (10%)