

115AT01

## 國立臺北科技大學 115 學年度碩士班招生考試

系所組別：1501、1502 自動化科技研究所

## 第一節 工程數學 試題

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**注意事項：**

1. 本試題共六題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1.  $A = \begin{bmatrix} a & f & e \\ f & b & d \\ e & d & c \end{bmatrix}$  is a  $3 \times 3$  symmetric real matrix. Its eigenvalues are 1, 4 and 9; the

corresponding eigenvectors are  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ,  $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ . Which of the following

statements are true (**this question must be answered with explanation**)? (15%)

(A)  $a = 5/2$ ; (B)  $b = d$ ; (C)  $c = 9$ ; (D)  $f = 0$ ; (E)  $e = 1$

2. Which of the following statements are true (**this question must be answered with explanation**)? (15%)

(A) Consider the  $4 \times 4$  orthogonal projection matrix  $Q = I - uu^T$ , where  $u \in \mathbb{R}^4$  and  $I$  is the  $4 \times 4$  identity matrix. Then,  $\det(Q) + \text{rank}(Q) = 4$ .

(B) Let  $A = \begin{bmatrix} 1 + \alpha_1 & \alpha_2 & \dots & \alpha_n \\ \alpha_1 & 1 + \alpha_2 & \dots & \alpha_n \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & \alpha_2 & \dots & 1 + \alpha_n \end{bmatrix}$ . Then,  $\det(A) = 1 + \sum_{i=1}^n \alpha_i$ .

(C) Let  $A$  be a square matrix. Also, let  $c$  and  $d$  be two column vectors. If  $Ax = c$ , then  $\det(A + cd^T) = \det(A)(1 + d^T x)$ .

(D) One can construct a  $3 \times 3$  Hermitian matrix  $A$  with complex-valued entries such that  $\det(A) = 1 + i$ , where  $i = \sqrt{-1}$ .

(E) None of the above

3. Solve the following initial-value problem. (15%)  
 $y' + 2x^3y = 0$ ,  $y(0) = 1$

4. Find the inverse Laplace transform  $y(t)$ . (20%)

$$Y(s) = \frac{s^3}{s^4 + 4}$$

5. Assuming that matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{bmatrix}$ , find the eigenvalues and eigenvectors of the matrix. (20%)

6. If matrix  $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ , kindly compute  $A^{100}$ . (15%)