114EE05

國立臺北科技大學114學年度碩士班招生考試

系所組別：2140 電機工程系碩士班丁組

第一節 線性代數 試題

注意事項：

1. 本試題共6題，第1題28分，第2題12分，第3題15分，第4題和第5題都10分，第6題25分，共100分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 所有答案均需詳細推導或說明，否則不予計分。

1. (28%) Consider the linear system

$$\begin{cases} x+y+2z+w=3 \\ -x+z+2w=1 \\ 2x+2y+w=-2 \\ x+y+2z+3w=5 \end{cases}$$

a. (4%) Define the coefficient matrix $\mathbf{A}$ for the linear system.

b. (4%) Find det($\mathbf{A}$).

c. (4%) Is the linear system consistent? Explain.

d. (4%) Are the column vectors of $\mathbf{A}$ linearly independent? Explain.

e. (4%) Find all solutions to $\mathbf{Ax}=\mathbf{0}$.

f. (4%) Is the matrix $\mathbf{A}$ invertible? If yes, then find the inverse.

g. (4%) Solve the linear system.

2. (12%) Please determine whether the following statements are true or false (need explanation).

a. (4%) Every 2×2 matrix can be written as a linear combination of

$\begin{bmatrix}1&0\\0&0\end{bmatrix}$, $\begin{bmatrix}0&1\\0&0\end{bmatrix}$, $\begin{bmatrix}0&0\\1&0\end{bmatrix}$

b. (4%) If $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are the column vectors of a 3×3 matrix $\mathbf{A}$, then the linear system $\mathbf{Ax}=\mathbf{b}$

has a unique solution for all vectors $\mathbf{b}$ in $\mathbb{R}^3$. $\mathbf{v}_1=\begin{bmatrix}1\\0\\1\end{bmatrix}$, $\mathbf{v}_2=\begin{bmatrix}2\\1\\0\end{bmatrix}$, $\mathbf{v}_3=\begin{bmatrix}4\\3\\-1\end{bmatrix}$,

c. (4%) The set of all linear combinations of matrices in S is equal to the set of all linear combinations of matrices in T.

$S=\{\mathbf{M}_1,\mathbf{M}_2,\mathbf{M}_3,\mathbf{M}_4\}$, $T=\{\mathbf{M}_1,\mathbf{M}_2,\mathbf{M}_3\}$,

$\mathbf{M}_1=\begin{bmatrix}1&-1\\0&0\end{bmatrix}$, $\mathbf{M}_2=\begin{bmatrix}0&0\\1&0\end{bmatrix}$, $\mathbf{M}_3=\begin{bmatrix}0&0\\0&1\end{bmatrix}$, $\mathbf{M}_4=\begin{bmatrix}2&-1\\1&3\end{bmatrix}$

3. (15%) Let $V=P_2$ with bases $B=\{1,x,x^2\}$ and $B'=\{1,x+1,x^2+x+1\}$. P_2 represents a polynomial of the degree 2.

a. (5%) Find the transition matrix $[I]_B^{B'}$ from B to B'.

b. (5%) Let $p(x)=3-x+2x^2$ and find $[p(x)]_{B'}$

c. (5%) Find the inverse of $[I]_B^{B'}$.

4. (10%) Let $T: P_2\to P_2$ be defined by $T(ax^2+bx+c)=ax^2+(a-2b)x+b$

a. (5%) Determine whether $p(x)=2x^2-4x+6$ is in the range of T, denoted by $R(T)$.

b. (5%) Find a basis for $R(T)$.

5. (10%) A group insurance plan allows three different options for participants, plan A, B, or C. Suppose that the percentages of the total number of participants enrolled in each plan are 25 percent, 30 percent, and 45 percent, respectively. Also, from past experience assume that participants change plans as shown in the table.

	A	B	C
A	0.75	0.25	0.2
B	0.15	0.45	0.4
C	0.1	0.3	0.4

a. (5%) Find the percent of participants enrolled in each plan after 2 years.

b. (5%) Find the steady-state vector for the system.

6. (25%) Find a singular value decomposition of the matrix (need complete procedures)

$$\mathbf{A}=\begin{bmatrix}1&1\\3&-3\end{bmatrix}$$