114CC02

國立臺北科技大學 114 學年度碩士班招生考試

系所組別：2220 電子工程系碩士班乙組

第一節　機率　試題

注意事項：

1. 本試題共七題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. True or False Questions: The following questions are all true or false questions. Please answer each question with either 'True' or 'False'.
   (a) Any random variable can be generated by a uniform distribution. (1%)
   (b) The inter-arrival time of events in a Poisson process follows an Exponential distribution. (1%)
   (c) The sum of two jointly Gaussian random variables is also Gaussian. (1%)
   (d) The variance of a random variable is strictly greater than 0. (1%)
   (e) The cumulative distribution function of a random variable X gives the probability that X takes on a specific value x. (1%)
   (f) The Geometric distribution is the only discrete probability distribution that has memoryless property. (1%)
   (g) The expectation of a random variable X always equals the median of X. (1%)
   (h) The probability of the union of two events A and B is always the product of their individual probabilities. (1%)
   (i) If an event has a probability of 0, it means the event is impossible and will never occur. (1%)
   (j) The Law of Large Numbers requires that the random variables are independent. (1%)

2. We cascade two function blocks of A in parallel for System 1 and two function blocks B in series for System 2, as illustrated in Figure 1 (a) and Figure 1 (b). The working probability of function block A is $P(A)$ and that of B is $P(B)$. Assume all function blocks fail independently
   (a) Given that $P(A) = P(B) = 0.9$, what are the probabilities that the two systems will work? (5%)
   (b) If we want to let both systems work with equal probability, what is the relationship between $P(A)$ and $P(B)$? Discuss which system needs a higher working probability for its function blocks. (10%)

(a) System 1　　(b) System 2

Figure 1: System Diagram

3. A single communication link with a capacity of 1 Gb/s is shared among N users. Each user requires 0.4 Gb/s for file transmission when active. Each user is active independently of the others, with a probability of being active equal to 20% (i.e., $P(\text{active}) = 0.2$). Answer the following questions:
   (a) If $N = 2$, what is the probability that at least one user is transmitting a file? (5%)
   (b) For $N = 3$, determine the expected capacity occupied by the active users. (5%)
   (c) The system is considered overloaded if the total required capacity exceeds 1 Gb/s. Determine the maximum number of users N that can be supported such that the probability of overload is less than 0.01. (10%)

4. A random variable X is uniformly distributed on the interval $[0, 5]$, with the probability density function:

$$f_X(x) = \begin{cases} \frac{1}{5}, & 0 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

   Define another random variable $Y = 2X + 1$.
   (a) Find the probability density function of Y. (5%)
   (b) Derive the conditional probability density function of X given that $Y \le 8$. (5%)

5. The number of days required to complete a communication experiment is represented by the random variable $Y = X + 20$, where X follows the probability density function:

$$f_X(x) = \begin{cases} \dfrac{24}{(x+2)^4}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Compute the expected number of days needed to complete the experiment. (10%)
   (b) Compute the variance of the number of days required to complete the experiment. (5%)

6. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables, each following an exponential distribution with the probability density function:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

   Define $M = \max\{X_1, X_2, \ldots, X_n\}$, the maximum value among the samples.
   (a) Derive the probability density function $f_M(m)$ of the maximum value M when $n = 2$. (5%)
   (b) Generalize and derive the probability density function $f_M(m)$ for the maximum value M for any positive integer n. (5%)
   (c) Compute the probability $P(M > 1)$ for any positive integer n. (5%)

注意：背面尚有試題

7. Define a random variable $W = X + Y + Z$, where:

- X is a Bernoulli random variable with: $P(X = 1) = 0.6, P(X = 0) = 0.4$.
- Y is a Gaussian random variable with mean 5 and variance 4.
- Z is conditionally dependent on X, and its distribution is described as:
  - $P(Z \mid X = 0)$ is a Gaussian distribution with mean 2 and variance 1,
  - $P(Z \mid X = 1)$ is a Gaussian distribution with mean 7 and variance 9.

Derive the explicit probability density function $f_W(w)$ of the random variable W. (15%)