114AT01

國立臺北科技大學 114 學年度碩士班招生考試

系所組別：1501、1502 自動化科技研究所

第一節　工程數學　試題

注意事項：

1. 本試題共 8 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Find the general solution of the following differential equation. (10%)

$$x\frac{dy}{dx} = y + x^3 + 3x^2 - 2x$$

2. Solve the following initial-value problem. (10%)

$$y' = \frac{2y}{x} + x^2e^x,\ y(2) = 0$$

3. Use the Laplace transform to solve the following initial-value problem. (20%)

$$y_1' + 2y_1 - 3y_2 = 0,\quad y_2' - 4y_1 + y_2 = 0$$
$$y_1(0) = 4,\quad y_2(0) = 3$$

4. For the following matrix A, find the inverse matrix A^{-1}. (5%)

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

5. Given $A^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \\ 0 & 1 & -2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -2 & 3 & 2 \end{bmatrix}$, find (AB^{-1}). (5%)

6. Assuming that matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, find **(a)** the eigenvalues (10%) and eigenvectors (10%) of the matrix and also find **(b)** a matrix P so $P^{-1}AP$ is a diagonal matrix (10%). (30% in total)

7. $A = \begin{bmatrix} a & f & e \\ f & b & d \\ e & d & c \end{bmatrix}$ is a 3×3 symmetric real matrix. Its eigenvalues are 1, 4 and 9; the corresponding eigenvectors are $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Which of the following statements are true **(this question must be answered with explanations)**? (15%)
(A) $a = 2$; (B) $a = b$; (C) $c = 9$; (D) $d = 0$; (E) $e = 1$

8. Which of the following statements is true? (5%)

(A) Let $A = \begin{bmatrix} 1+\alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \alpha_1 & 1+\alpha_2 & \cdots & \alpha_n \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & \alpha_2 & \cdots & 1+\alpha_n \end{bmatrix}$. Then, $\det(A) = 1 + n\sum_{i=1}^{n}\alpha_n$.

(B) Let A be a square matrix. Also, let c and d be two column vectors. If $Ax = c$, then $\det(A + cd^T) = \det(A)(1 + d^Tx)$.

(C) Consider the 4×4 orthogonal projection matrix $Q = I - uu^T$, where $u \in \mathbb{R}^4$ and I is the 4×4 identity matrix. Then, $\det(Q) + \text{rank}(Q) = 4$.

(D) One can construct a 3×3 Hermitian matrix A with complex-valued entries such that $\det(A) = 1 + i$, where $i = \sqrt{-1}$.

(E) None of the above