國立臺北科技大學100學年度碩士班招生考試

系所組別：2140、2150 電機工程系碩士班丁、戊組

第二節　工程數學　試題

第一頁　共一頁

注意事項：
1. 本試題共7題，配分共100分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (12%) A coin is biased so that a head is twice as likely to occur as a tail. Toss the coin 3 times. Event A denotes that at least 2 heads occur in three tosses. Event B denotes that only one tail occurs in three tosses. Find $P(A)$, $P(B)$, and $P(B/A)$.

2. X and Y are two random variables with joint probability density function

$$f_{XY}(x,y)=\begin{cases}1 & 0\le x\le 1, 0\le y\le 1\\ 0 & \text{else}\end{cases}.$$

Define the random variables $U=X+Y$ and $V=X-Y$.

a) (4%) Find the joint cumulative distribution function $F_{XY}(x,y)$.

b) (4%) Find the cumulative distribution functions $F_X(x)$ and $F_Y(y)$.

c) (2%) Are X and Y independent? Explain your answer.

d) (4%) Find the value of the joint cumulative distribution function $F_{UV}(1,0)$.

e) (4%) Find the cumulative distribution function $F_U(u)$ of U.

f) (2%) Find the probability density function $f_U(u)$ of U

3. $X_1, X_2, \cdots$ are independent random variables with the same probability mass function $\mathrm{P}(X_n=0)=\mathrm{P}(X_n=2)=\frac{1}{2}, n=1,2,\cdots$. Define the random variables Y_n , $n=1,2,\cdots$ by $Y_n=\sum_{k=1}^{n}X_k$.

a) (4%) Find EX_n and $\mathrm{var}(X_n)$.

b) (4%) Find EY_n and $\mathrm{var}(Y_n)$.

c) (3%) Find the covariance function $\mathrm{cov}(Y_i,Y_j)$ for $i,j\in\mathbb{N}$.

d) (3%) Find the probability mass function of Y_n.

e) (4%) Find $\mathrm{P}(Y_n=i, Y_{n+1}=j)$.

4. (10%) Let $A=\begin{bmatrix}1&1&2&0\\2&1&3&1\\1&2&0&1\end{bmatrix}$. Find the nullity and the null space of A.

5. (10%) Let L be the linear mapping in R^3 defined by $L(x)=Ax$ corresponding to the standard basis, where $A=\begin{bmatrix}3&-1&-2\\2&0&-2\\2&-1&-1\end{bmatrix}$, and let $v_1=\begin{bmatrix}1\\1\\0\end{bmatrix}$, $v_2=\begin{bmatrix}1\\0\\1\end{bmatrix}$, and $v_3=\begin{bmatrix}0\\1\\1\end{bmatrix}$ form another basis $[v_1,v_2,v_3]$. Find the matrix B representing L with respect to $[v_1,v_2,v_3]$.

6. Let $A=\begin{bmatrix}1&0&2&1\\2&0&1&1\\2&7&7&1\\2&1&7&1\end{bmatrix}$

a) (10%) Find the determinant and all eigenvalues of A;

b) (10%) Find the inverse of A.

7. (10%) Prove in English that "The system of n linear equations in n unknowns $Ax=b$ has a unique solution of and only if A is nonsingular.".

(Note : No credit will be given if the answer is given in Chinese)