113 VE 01

國立臺北科技大學 113 學年度碩士班招生考試

系所組別：1301、1302、1303 車輛工程系碩士班

第一節　工程數學　試題

注意事項：

1. 本試題共六題，每題配分如題示，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、Consider the first-order differential equation $y' = -\frac{4x}{y}$.

1. (10%) Find the general solution.
2. (10%) Find the solution for the initial condition $y(2) = 3$.

二、Consider the second-order differential equation $y'' + y = \cos x - \sin x$.

1. (5%) Find the general solution of the homogeneous ordinary differential equation (ODE).
2. (5%) Find the particular solution.
3. (5%) Find the general solution of the nonhomogeneous ODE.

三、Consider the third-order differential equation $y''' - 3y'' + 3y' - y = e^x/x$.

1. (5%) Find the general solution of the homogeneous ordinary differential equation (ODE).
2. (5%) Find the particular solution.
3. (5%) Find the general solution of the non-homogeneous ODE.

四、1. Find the Laplace transform of $f(t) = \cos(\omega t + \theta)$. (10%)

2. Find the inverse Laplace transform of $F(s) = \frac{s+1}{s^2+9}$. (10%)

(Hint: $\mathcal{L}\{\sin \omega t\} = \frac{\omega}{s^2+\omega^2}$, $\mathcal{L}\{\cos \omega t\} = \frac{s}{s^2+\omega^2}$)

五、1. Find the inverse of $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$. (10%)

2. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. (10%)

六、Find the Fourier transform $\mathcal{F}(e^{-ax})$ of $f(x) = e^{-ax}$ if $x > 0$ and $f(x) = 0$ if $x < 0$; here $a > 0$. (10%)