113MT01

國立臺北科技大學113學年度碩士班招生考試

系所組別：1201 製造科技研究所

第一節 微分方程 試題（選考）

注意事項：

1. 本試題共6題，每題15-25分，共100分。
2. 不必抄題，作答時請將試題題號及答案依照順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve for the following differential equation: (15 分)

$$y' = y^2 e^{-x}$$

2. Solve for the following differential equation: (15 分)

$$(\cos(x) - 2xy)dx + (e^y - x^2)dy = 0$$

3. Solve for the following differential equation: (15 分)

$$y' + \frac{1}{x}y = \frac{2}{x^3}y^{\frac{-4}{3}}$$

4. Solve for the following differential equation using the method of undetermined coefficient: (15 分)

$$y'' + 2y' + 2y = 3.5\sin 3x - 3\cos 3x$$
$$\text{and} \quad y(0) = 0 \quad y'(0) = 0.5$$

5. Use the Laplace transform to solve the following equation: (15 分)

$$y'' + 4y = f(t),\ y(0) = y'(0) = 0, f(t) = \begin{cases} 0, & t < 3 \\ t, & t \geq 3 \end{cases}$$

6. Consider the temperature distribution $u(x,t)$ along a thin, homogeneous bar of length L. The initial temperature function $f(x)$ is a constant A. The both ends of the bar are kept at a zero temperature. The governing equation and boundary conditions are:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \text{ for } 0 < x < L, t > 0$$
$$u(0,t) = u(L,t) = 0, t \geq 0$$
$$u(x,0) = f(x) = A$$

Please step-by-step solve for the function of the temperature distribution $u(x,t)$ (25 分)