

國立臺北科技大學 113 學年度碩士班招生考試

系所組別：2140 電機工程系碩士班丁組

第一節 線性代數 試題

第 1 頁 共 1 頁

注意事項：

1. 本試題共 6 題，共 100 分。
2. 不必抄題，作答時請將試題題號及答案依照題號順序寫在答案卷上。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 所有答案均需詳細推導或說明（包含使用之數學符號必須清楚定義），字體必須清晰可辨識，否則將酌予扣分。

1. (15%) Find the least-squares solution of $\mathbf{Ax} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

2. (25%) Let $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}$ be the eigenvectors that correspond to distinct eigenvalues a, b, c, d of an $N \times N$ matrix \mathbf{A} .
- (1) (10%) Show that the eigenspace of \mathbf{A} corresponding to a is a subspace.
 - (2) (15%) Show that the vector set $\{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}\}$ is a linearly independent set.
3. (10%) Rank Theorem: For an $N \times N$ matrix \mathbf{A} , we have $\text{rank}(\mathbf{A}) + \text{nullity}(\mathbf{A}) = N$. Use the theorem to show that $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution when \mathbf{A} is of full rank.
4. (20%) Consider two square matrices \mathbf{A} and \mathbf{B} where \mathbf{A} and \mathbf{B} are similar.
- (1) (10%) Show that \mathbf{A}^2 is similar to \mathbf{B}^2 .
 - (2) (10%) Show that if \mathbf{A} is diagonalizable, then \mathbf{B} is also diagonalizable.

5. (20%) Let \mathbf{U} be an orthogonal matrix and define \mathbf{H} as

$$\mathbf{H} = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

- (1) (5%) Find the eigenvalues and their multiplicities for \mathbf{H} .
 - (2) (5%) Find $\det(\mathbf{U}\mathbf{H}\mathbf{U}^T)$.
 - (3) (5%) Find $\text{trace}(\mathbf{U}\mathbf{H}\mathbf{U}^T)$.
 - (4) (5%) Find the norm of $\mathbf{U}\mathbf{H}\mathbf{x}$ if $\mathbf{H}\mathbf{x} = [a \ b \ c \ d]^T$.
6. (10%) Prove or disprove $\det(\mathbf{X} + \mathbf{Y}) = \det(\mathbf{X}) + \det(\mathbf{Y})$ for any square matrices \mathbf{X} and \mathbf{Y} .